Mrezni modeli prostornih podataka (Network Models in GIS)

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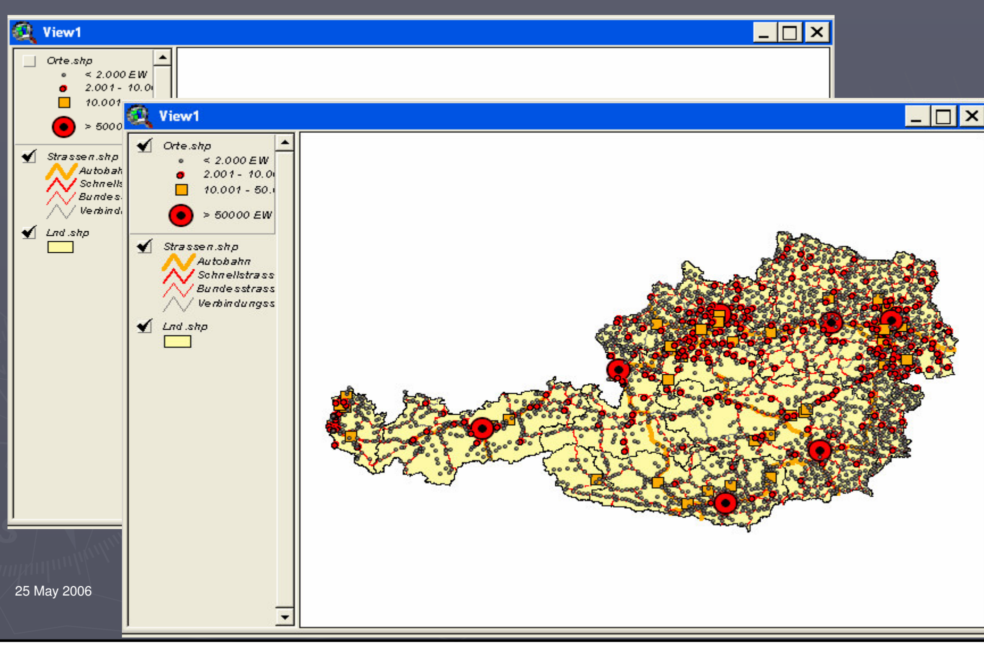
Example 1: Austrian road map



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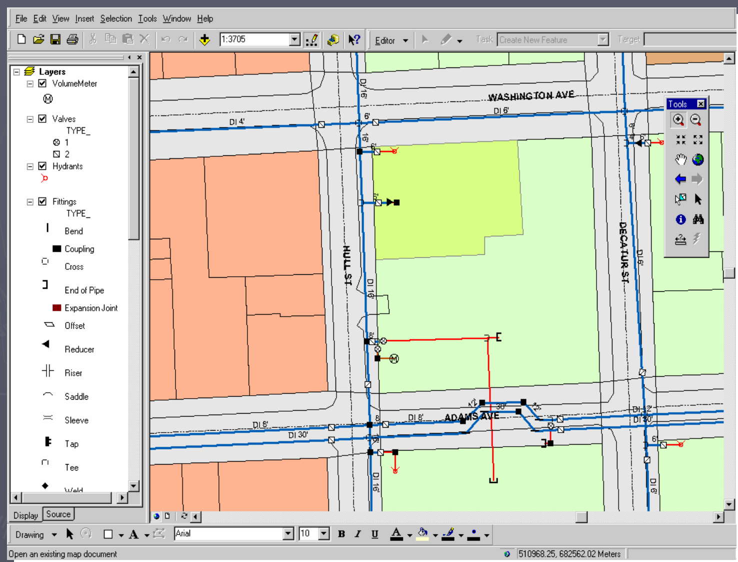
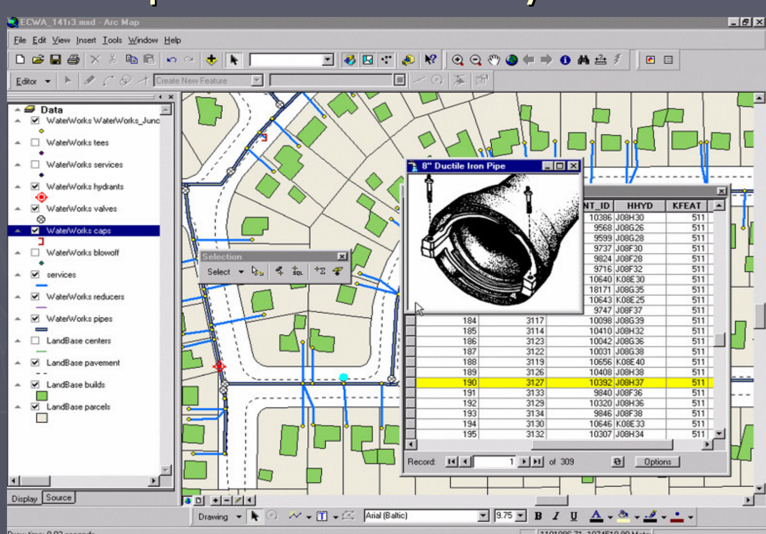
Example 2: Electrical utility application of GIS



showing
a part of a
customer record,
a detailed map of
connections in a
neighborhood, etc.

(Longley et al. 2001, p.28)

Example 3: Water Utility



GIS Application
for managing
the assets of a
water utility

Longley et al., 2001, p.28)

Logistics

- Many applications of GIS in transportation and logistics, e.g.
  - Infrastructure maintenance
  - Travel planning
  - Routing
  - Vehicle tracking
- Two components
  - Static – infrastructure
  - Dynamic - vehicles

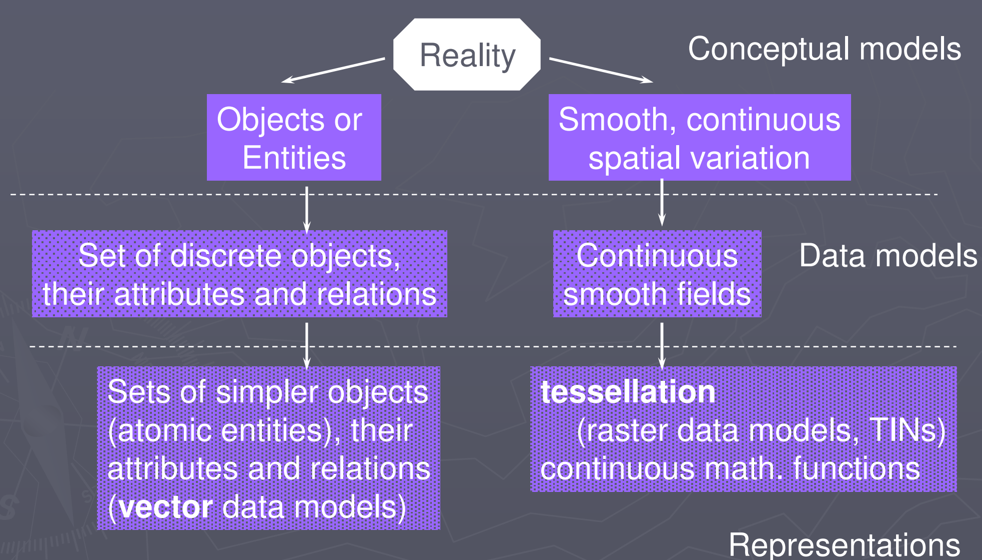
Planning for Emergency Evacuation

- Major natural and human-induced events may necessitate area evacuations
- GIS can be used to create effective evacuation vulnerability maps based on
  - Distribution of population
  - Street map
- Model demand and impact of bottlenecks on speed of evacuation using standard GIS network tools
  - Adjacency, connectivity, shortest path network calculation

Contents

- Spatial data modelling
- Fundamental spatial concepts
  - Geometry and Topology
  - Metric spaces
  - Graph theory
  - Network spaces
- Applications of network models

Spatial data concepts and models



Five Ms of Applied GIS

- Mapping
- Measuring
- Monitoring
- Modeling
- Managing

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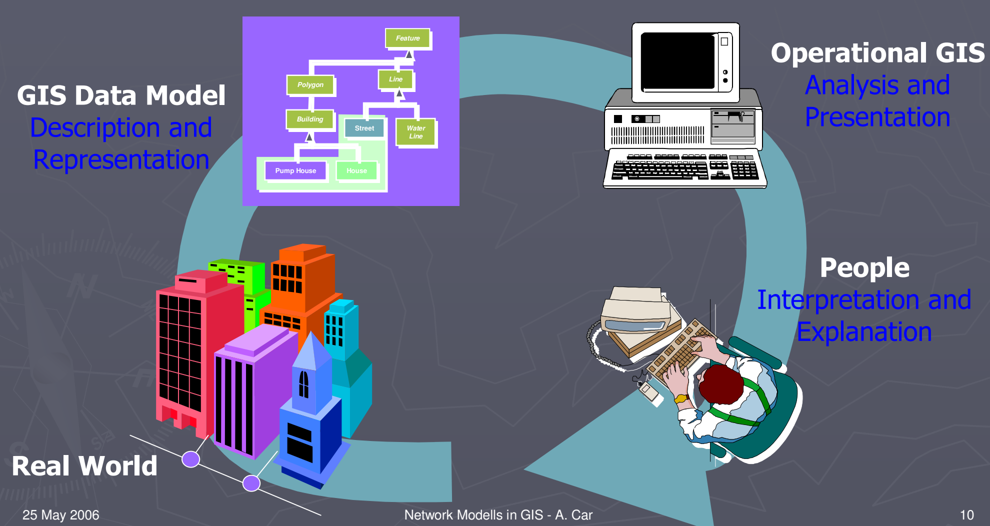
GeoData Modeling



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Fundamental spatial concepts

- Spatial data models formally describe the meaning (semantics) of spatial concepts
- Formal description is necessary, because computer systems are essentially formal systems (i.e. they manipulate symbols according to formal rules)

  Spatial object = attributes + geometry + time
- Well-defined mathematical concepts such as metric, topology and order
  are used to solve geometrical problems

Geometrical questions

- How far is it from Villach to Zagreb?
- How long is the river Sava?
- In which direction lies Bedekovcina?
- What is the area of Hrvatsko Zagorje?
- Which slope on Sljeme is the most suitable for a snowboard half-pipe?
- Do Austria and Croatia cover equally big areas?
- ...

Geometry (1)

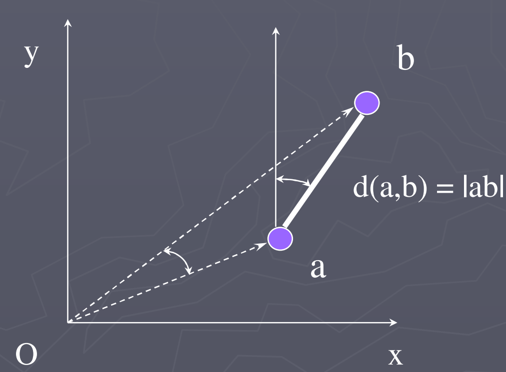
Geometry provides formalisms which represent abstract properties of structures within the space

- Geometry (Greek) = measurement of the Earth

- Erlangen program (Klein, 1872)
  - the beginning of modern treatment of geometry
  - founded on notion of invariance:
    geometries are distinguished by the group of transformations of space under which their propositions remain true

Geometry (2)

- **Euclidean geometry** – widely used in GIS
- embedding in a coordinated space, which enables measurements of
  - distance (“as the crow flies”)
  - angle
  - bearing (azimuth)
- transforms spatial properties such as
  length, area, shape, volume, gradient, …,
  into tuples of real numbers



Topology

- Topology (Greek) = study of form
- **Topology** is the science and mathematics of relationships used
  - To validate the geometry of entities (vector model)
  - In operations such as network tracing or test of polygon adjacency
- **Topological properties** are based on a non-metric information
  - Connectivity (e.g. roads)
  - Orientation (from, to)
  - Adjacency (sharing common boundary)
  - Proximity (closeness)
  - Containment (e.g. a city is within a region)
- …is one which is preserved by topological transformations of the space

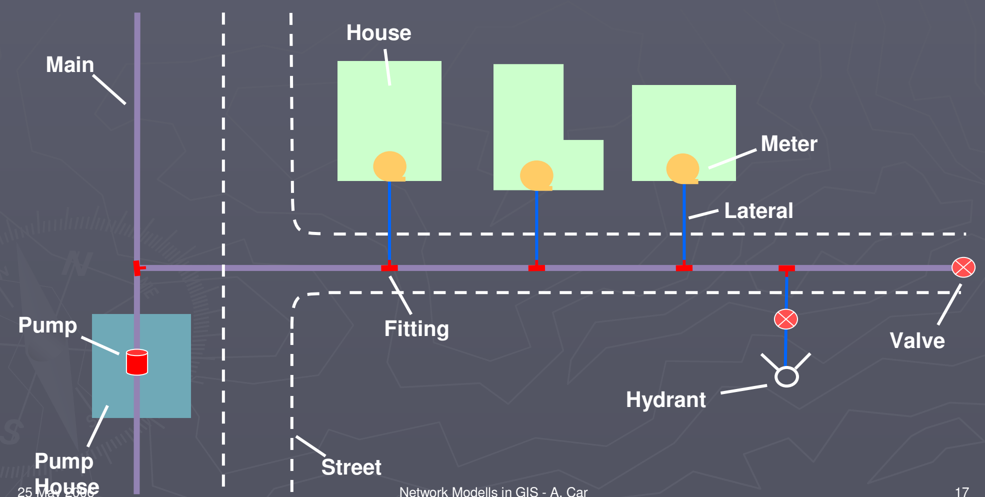
Topology (2)

Why use topologically structured data?
=> to improve the spatial analysis in GIS

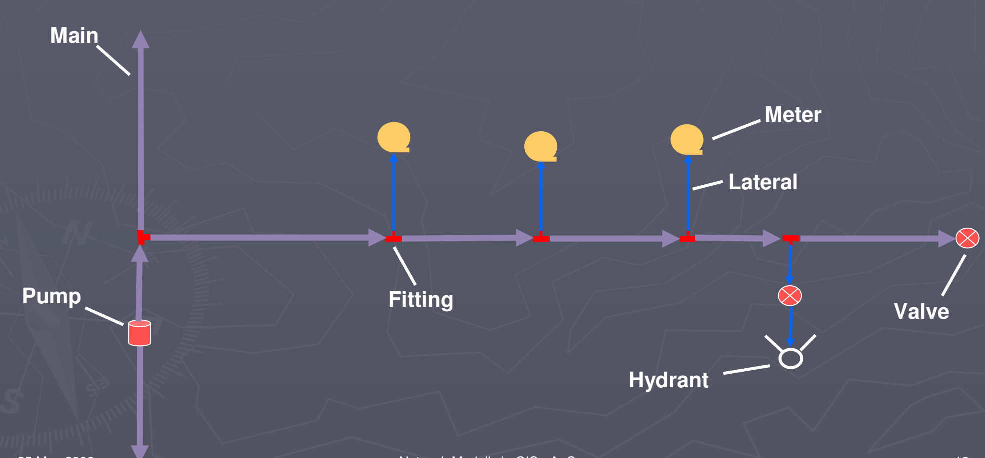
Examples:

- an area can be computed only for a closed polygon
- length of a river is a sum of lengths of its segments
- All lines in a network are interconnected
- …

Water Facility Data Types



Topological Network Model



Metric Spaces (1)

- Metric spaces include the concept of distance between objects in space.
- A **pointset** S is said to be a metric space if there exists a function, **distance**, that takes ordered pairs (s, t) of elements of S and returns a real number **distance (s, t)** that satisfies the following conditions:

Metric Spaces (2)

- For each pair (s,t) in S,
  distance (s,t) > 0 if s and t are distinct points,and
  distance (s,t) = 0 if s and t are identical
- For each pair (s,t) in S, the distance from s to t is equal to the distance from t to s,
  distance (s,t) = distance (t,s)
- For each triple s,t,u in S, the sum of the distances from s to t and from t to u is always at least as large as the distance from s to u, that is:
  distance (s,t) + distance (t,u) ≥ distance (s,u)

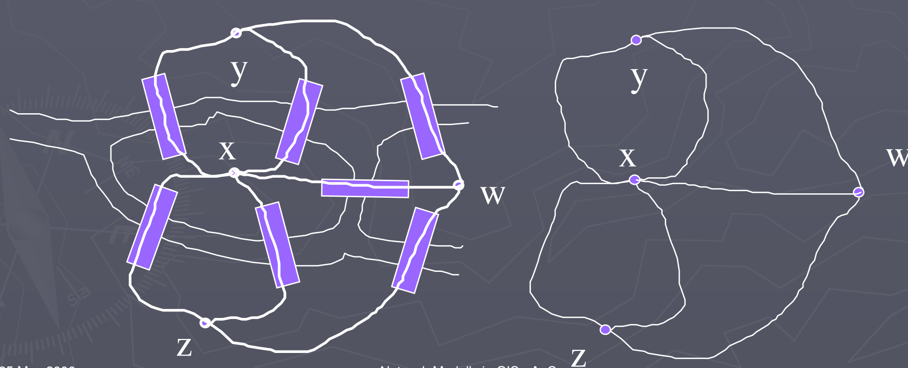
Metric Spaces (3)

Some possible distance functions
e.g. between the two city centers on the globe:

- **Geodesic distance**: "as the crow flies", i.e. distance along the great circle of the Earth passing through the 2 city centers
- **Spherical Manhattan distance**: the difference in their latitudes + difference in their longitudes
- Traveling time: the min. time that is possible to travel from one city to the other using a sequence of scheduled airline flights
- **Lexicographic**: the absolute value of the difference between their positions in a list of cities in a fixed gazetteer

Network Spaces (1)

- Leonard **Euler**, mathematician, who in 1736 solved the problem of Koenigsberger Bridges;
  the founder of the systematic study of topology



Network Spaces (2)

- Used to represent many different geographic problems, e.g.
  - Transportation, logistics, emergency management, hydrology, geomarketing, AM/FM, …
- Modelling and calculations based on a **graph** as a formal model of the network

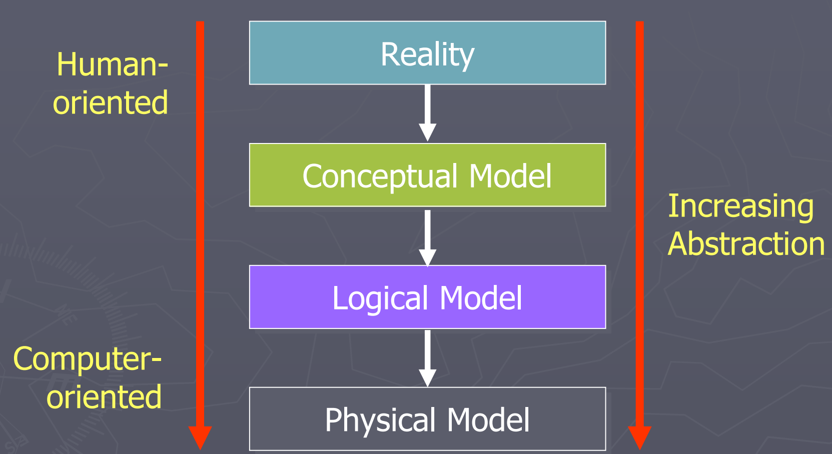
⇒ **Graph theory** is the focus of this lecture

Abstract graph

Network representation

Dijkstra algorithm

Data Model Levels

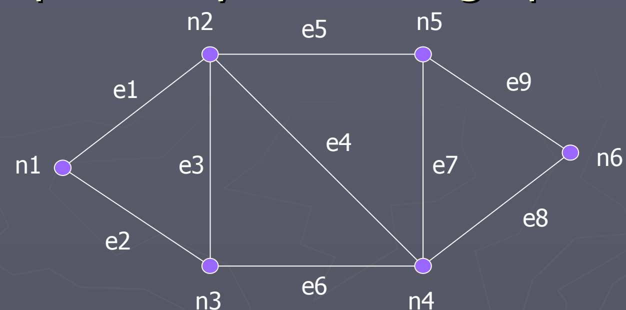


Graph theory: Abstract graphs (1)

- **Graph *G*** – a non-empty set of **nodes** (N) together with a set of unordered pairs of distinct nodes called **edges** (E) (also called *arc*, *network link*)

ni, nj are nodes of *G* and *e=ni nj* is an egde of *G*, then
e joins *ni* to *nj* or is incident with *ni* and *nj*
ni and *ynj* are incident with *e*

Graph theory: Abstract graphs (2)



• Finite graph – if both the number of nodes and the number of edges are finite
• In a complete graph each pair of distinct nodes defines an edge

Graph theory: Abstract graphs (3)

- A self-loop is an edge (ni,nj) for which ni=nj
- Parallel edges have end-nodes in common, to distinguish them, attributes like travel time or direction are necessary

- A simple graph
  contains no parallel edges or self-loops
- A multi-graph
  contains parallel edges but no self-loops

Graph theory: Abstract graphs (4)

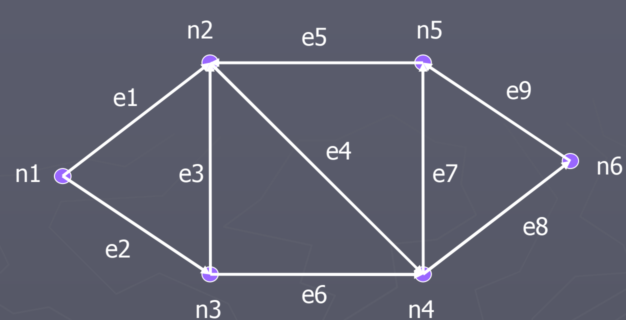
- Dense graphs - graphs where only few edges are missing
- Sparse graphs – graphs with relatively few edges

- Planar graph - if it can be drawn in a 2D (Euclidean) plane so that no two edges cross or intersect each other, i.e., edges can meet only at nodes

Graph theory: Abstract graphs (5)

- Graph is a highly abstracted model of spatial relationships and represents only connectedness
- Possible extensions: directed and labeled graphs

- **Directed graph (digraph)**
  - has directions assigned to its edges
  - represented as arrowed lines
  - E.g. one-way streets

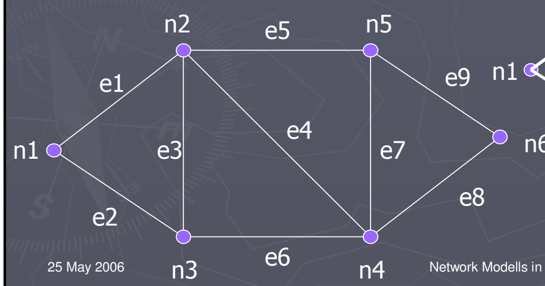


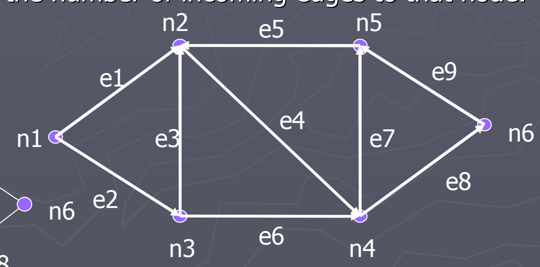
- A bidirectional graph (or symmetric graph) is a digraph in which ∀ edge (ni,nj) ∃ edge (nj,ni)

- A undirected graph - directions of edges are removed

Graph theory: Abstract graphs (7)

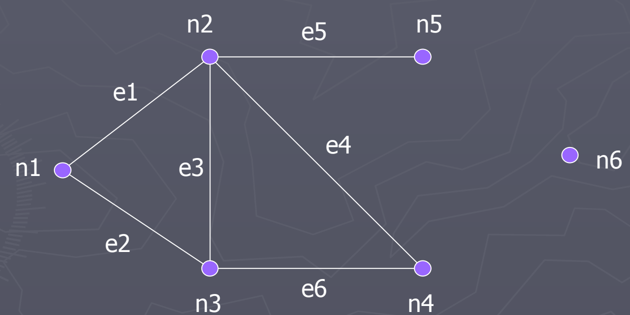
- The number of edges meeting in a node is the degree of that node.
  - In a digraph the outdegree of a node is the number of edges leaving the node,
  - and the indegree of a node is the number of incoming edges to that node.





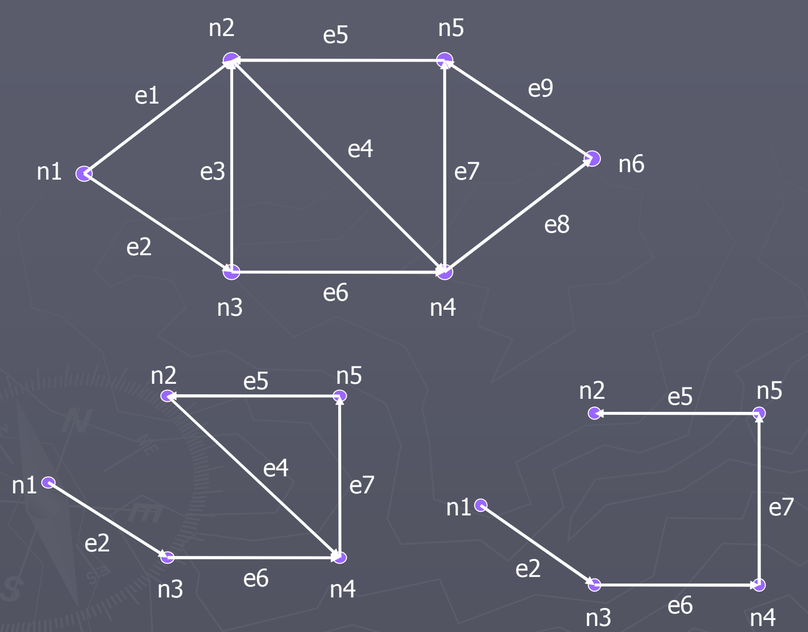
Graph theory: Abstract graphs (8)

- If a certain number of edges and/or nodes is removed from a graph, we obtain its subgraph
  - Problem of isolated nodes



Graph theory: Abstract graphs (9)

- A walk in a graph is a sequence of succeeding nodes
- A path is a walk without any repetition of nodes
- A cycle is a walk where the first node of the first edge coincides with the last node of the last edge

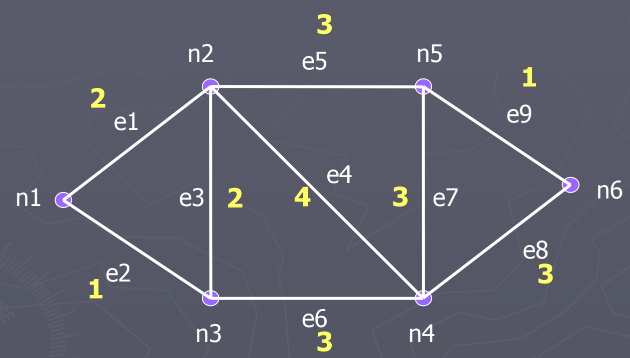


Graph theory: Abstract graphs (6)

- Connected graph
  - A graph G is connected if any two nodes of G are connected by a path.
  - A connected graph is strongly connected if there is at least one directed path from every node to every other node.
- Acyclic graph – a graph that has no cycles
- Tree – a connected acyclic graph
  - Rooted tree, leaf
  - Directed acyclic graph (DAG)

Graph theory: Abstract graphs (7)

- **Labeled graph** (weighted graph)
  - each egde is assigned a label (a number or string),
  - often indicated near the appropriate edge
  - E.g. travel time or distance
- A path with the minimum weight is called **the shortest path**
  - nodes are labeled with tentative distances at each step: *distance labels* are estimates of the shortest path distances

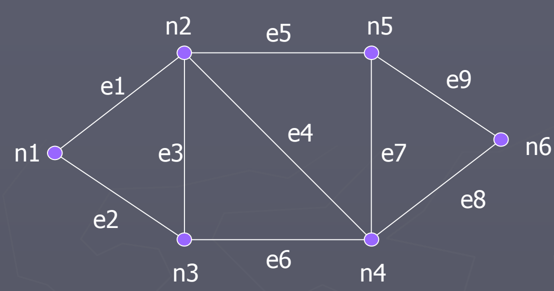


Network representation (1)

- There are different ways to represent graphs suitable for computational purposes
  - use matrices because they capture incidence relations between nodes and edges well
- The *adjacency matrix* A(G) of the graph G
  - an n x n matrix, where n is the number of nodes in G
  - aij ∈ A(G)

    aij=1 if there exists an edge from ni to nj;

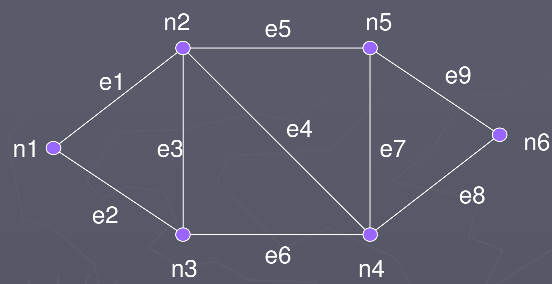
    aij=0 if there is no edge from ni to nj.
  - The matrix is symmetric if a graph is undirected



$$A(G) = \begin{pmatrix} 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

Network representation (2)

- The incidence matrix B(G) of an undirected graph
  - is an n x m matrix of 0's and 1's, where
    n is a number of nodes and m is a number of edges.
    - bij=1 if the edge ej meets the node ni, and
      bij =0 if not
  - In the case of digraph we distinguish between incoming and outgoing edges of a node:
    - bij =+1 if ej starts at ni, bij =-1 if ej ends in ni, and
      bij =0 otherwise.



$$A(G) = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

An example:
Incidence matrix A(G)

Dijkstra's algorithm (1)

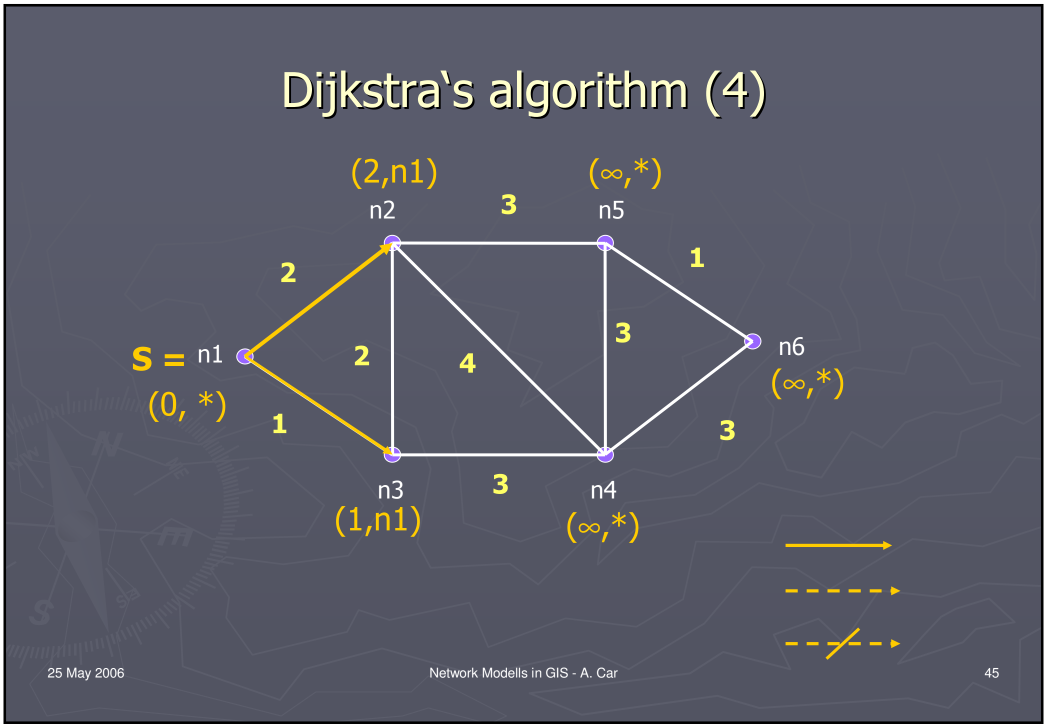
- *Dijkstra's algorithm* is the best known for solving shortest path problems.
- It finds the shortest paths from the start node to all other nodes in a network with nonnegative edge lengths
  - also known as *single-source shortest path* problem

Dijkstra's algorithm (2)

- Input:
  - G=(N,E) undirected simple connected graph
  - s ∈ N starting node
  - w : E → R+ weighting function (edge weights)
  - t : N → R+ target weighting function
    (stores min distances from s to each node in the graph)
- Output:
  - t : N → R+ Graph weights

Dijkstra's algorithm (3)

1. Initialize t(n) ← ∞ ∀ n∈N, visited node set V←{s}
2. Set t(s) ← 0
3. **For all** n∈N such that edge sn ∈E **do**
4. Set t(n) ← w(sn)
5. **While** N≠V **do**
6. find by sorting n∈N\V such that t(n) is minimised
7. add n to V
8. **for all** m∈N\V such that edge nm ∈E **do**
9. t(m) ← min(t(m), t(n)+w(nm))


Dijkstra's algorithm (4)
(2,n1)
n2
3
(∞,*)
n5
2
1
S = n1
2
4
3
n6
(0, *)
(∞,*)
1
3
n3
3
n4
(1,n1)
(∞,*)
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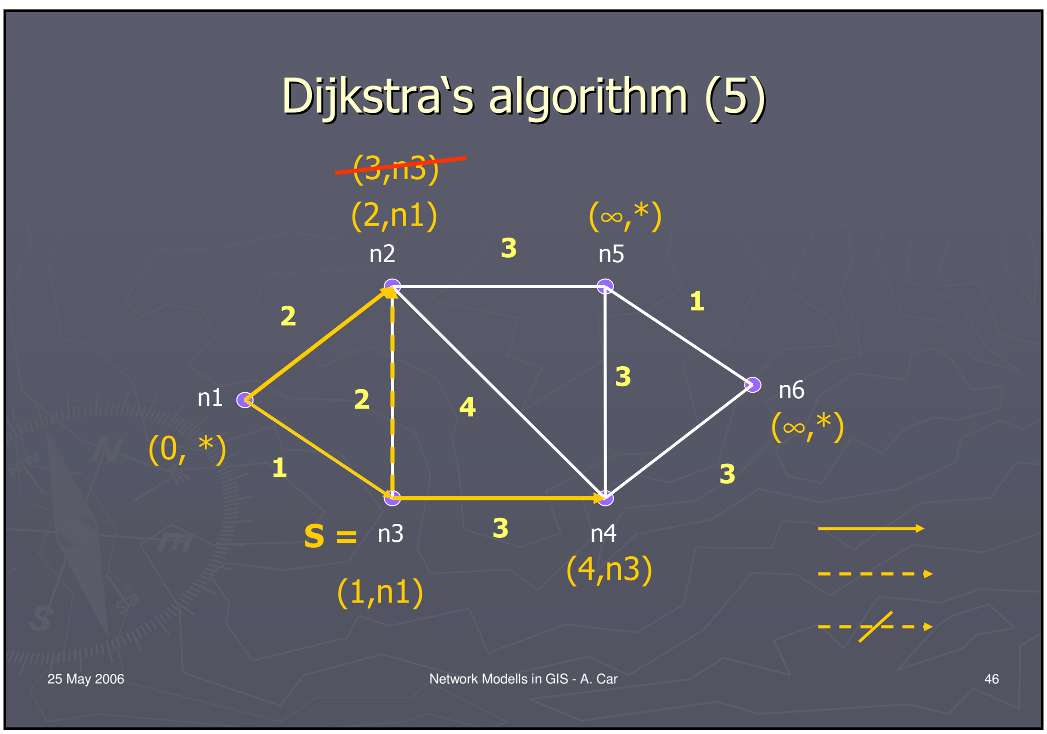
Dijkstra's algorithm (5)
(3,n3)
(2,n1)
n2
3
(∞,*)
n5
2
1
n1
2
4
3
n6
(0, *)
(∞,*)
1
3
S = n3
3
n4
(4,n3)
(1,n1)
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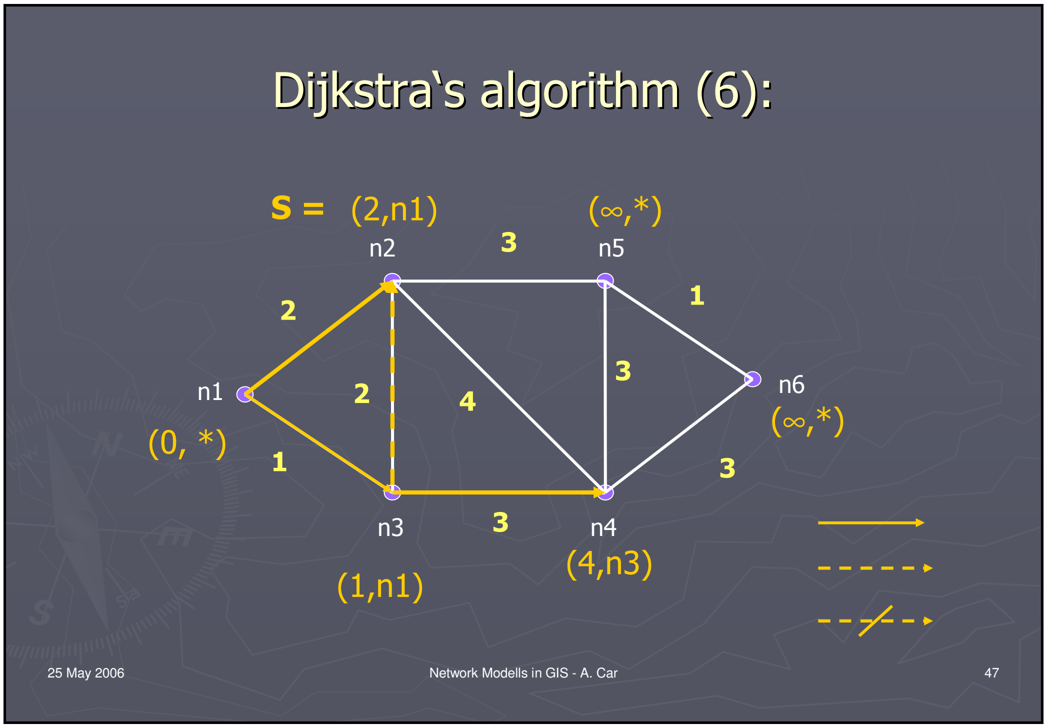
Dijkstra's algorithm (6):
S = (2,n1)
(∞,*)
n2
3
n5
2
1
n1
2
4
3
n6
(∞,*)
(0, *)
1
3
n3
3
n4
(4,n3)
(1,n1)
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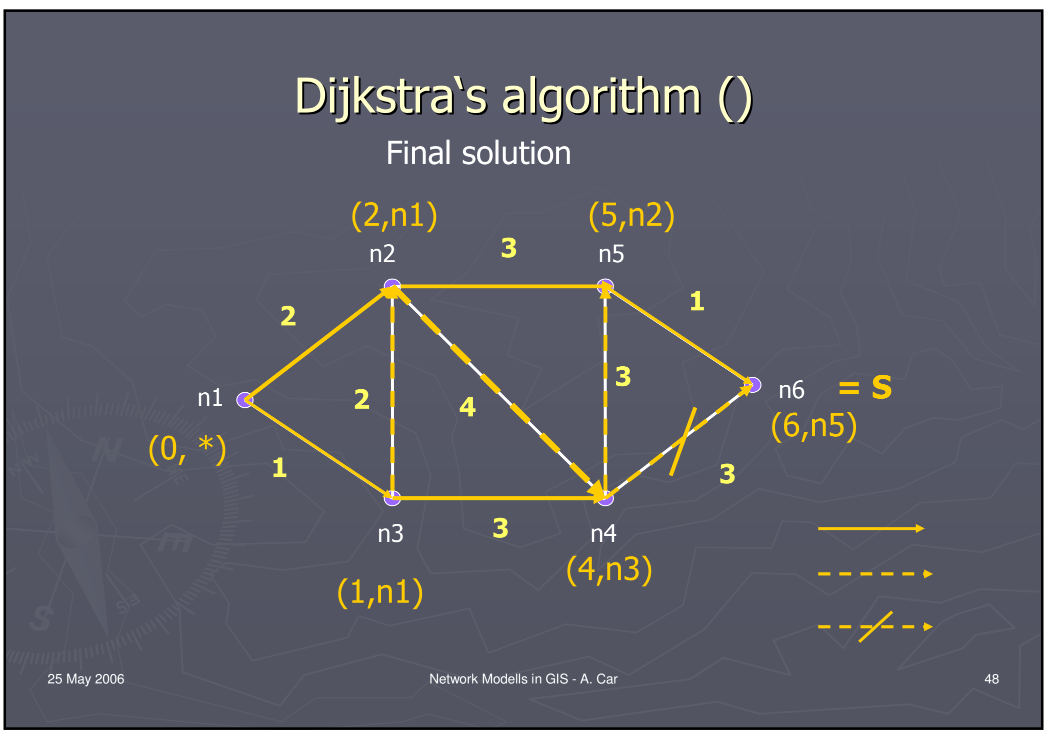
Dijkstra's algorithm ()
Final solution
(2,n1)
(5,n2)
n2
3
n5
2
1
n1
2
4
3
n6 = S
(6,n5)
(0, *)
1
3
n3
3
n4
(4,n3)
(1,n1)
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Network models in GIS (1)

Network operations

- Pathfinding - find the shortest, least cost, or most efficient path on a network.
- Tracing - determine a connected portion of a network that are either flow from this connected portion of the network to a given node or flow from a given node to this connected portion of the network.
- Allocation - assign portions of a network to a location (e.g., a center) based on some given criteria.

Network models in GIS (2)

Network applications

- *Geocoding* - building a relationship between locational data in a database and street address data that are normally in a tabular format
- *Location-allocation* - determing the optimal locations for a given number of facilities based on some criteria and simultaneously assigning the population to the facilities

Network models in GIS (3)

Network applications

- *Business logistics* is concerned with the optimization of vehicle routing and delivery scheduling
- Spatial interaction and gravity modeling
  The interaction between different locations in geographic space and the mathematical modeling of the interaction are important in application areas such as transportation and retail analyses. Gravity models are commonly used to support these analyses.
- Dynamic segmentation is a particular network model used to represent, analyze, query, and display linear features. The basic difference between dynamic segmentation and the network representations discussed above is that dynamic segmentation has the flexibility to associate an attribute to a portion of an arc or several arcs

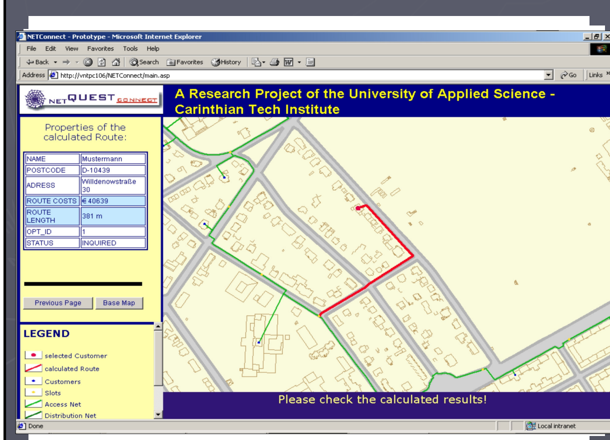
Network applications in GIS (4)

Evacuation vulnerability map of the area of Santa Barbara, Ca, USA

(Longley et al., 2001, p.49)

Network applications in GIS (5)

Optimization of optical networks considering geoinformation



- Develop a **spatially balanced score card**
  - formalized expert knowledge
  - base for a cost function used in optimization
- Approach
  - Knowledge engineering methods (scenarios)
  - involves principles of HSR

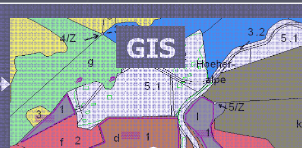
=> **Design a SDSS**

Modeling Forest Road Networks

Problem:

- Planning a forest road is extremly time consuming (exploring the area on foot)
- No optimal solution possible when calculating by hand (no alternatives calculated, suboptimal use of geodata)

ÖBf Österreichische Bundesforste AG



Operations Research

Mathematical Modeling

$C = f\,(x1, x2, \ldots xn)$

Spatial Decision Support System

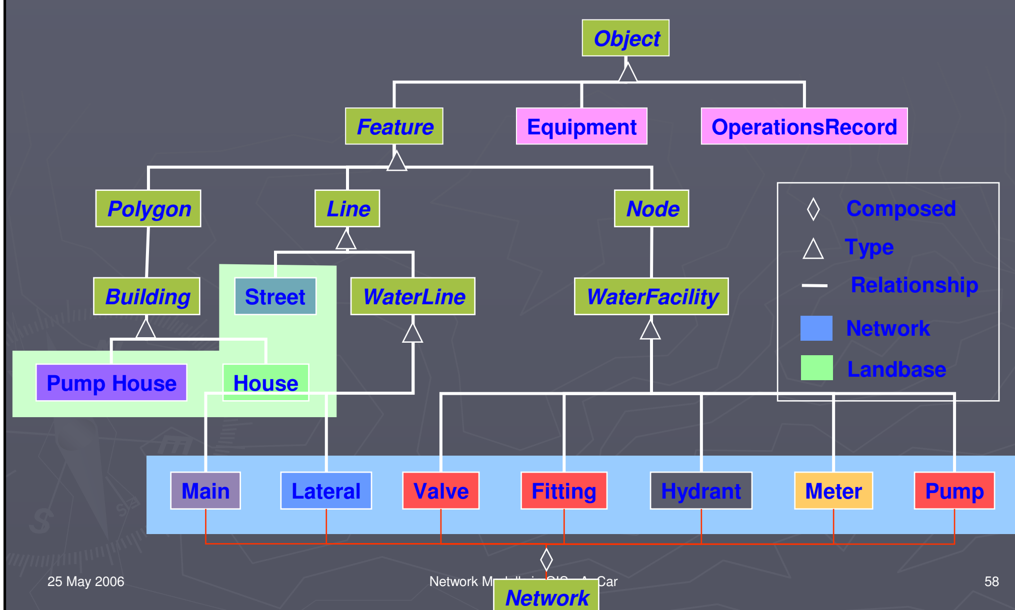
Fundamental spatial concepts Summary

- Geometry and tolopogy are necessary for query and analysis of spatial data
- Geoinformation has geometric and topological properties
- „Geometry-Toolbox“ includes sets, relations, functions, graphs
- GIS uses different spaces: e.g., Euclidean, metric, or network spaces

Example Water Facilities Data Model

- Start with objects and relationships
- Model as object types and relationships
  - Topological network
  - Hierarchical 'type of'
  - Collection 'composed of'
- Add related attribute tables

Water Facility Object Model



Literature (1)

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Literature (2)

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Terms and Definitions (1)

- Data model
  - Abstract description of reality
  - Set of constructs for representing objects and processes in the digital environment
- Geographical (spatial) data model
  abstraction and representation of spatial phenomena according to a formalized concept, which is usually implemented using geometric primitives of points, lines, areas or discretized continuous fields
- Representation
  - Focus on conceptual and scientific issues

Terms and Definitions (2)

- Cognition - acquisition and use of knowledge by an individual
- Spatial cognition - processes by which spatial knowledge is acquired, stored and recalled
- Conceptual model of space / spatial concept
  objects and spatial relations arranged according to our experience and cognition