AN ARTISTIC APPROACH TO THE TESSERACT

„Es ist die Freude an der Gestalt in einem hoheren Sinne, die den Geometer ausmacht"

Clebsch, in memory of J. Plücker

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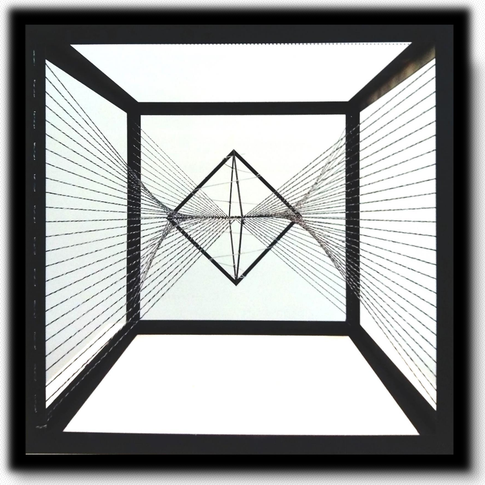
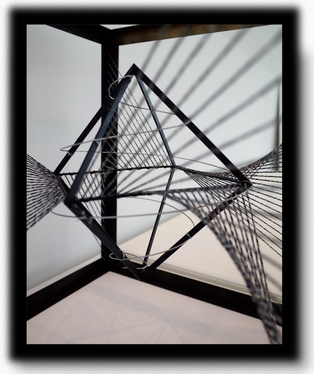




The string model is a homage to the first material mathematical models used to study ruled surfaces in the 19th century and the geometrical structure of the model is a homage to number 4 representing a tesseract.
The outer frame is a cube, Platonic solid with 8 vertices and the inner object is an octahedron,
a Platonic solid dual to the cube, with 8 edges.
They are connected with 4 hyperbolic paraboloids,
each one containing 1 edge of the cube and 1 edge of the octahedron.
Hyperbolic paraboloids are 2nd degree ruled surfaces with 2 system of rulings.
The strings in the model represent one system of rulings and the 2 edges of the tesseract belong to the other one.
Octahedron is decorated with 2 spirals which pass through 3 vertices of the hypercube.

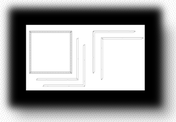
The frame of the tesseract is made from HDF panels cut by a laser cutter.
The cube is made from 24 pieces and the octahedron from 5, glued and colored. The strings are recycled wool.
The spirals are made of one piece of steel wire.

This project started in September 2018 at the international summer school Line Geometry for Lightweight Structures organized by prof. dr. sc. Daniel Lordick at TU Dresden for students, doctoral students and postdocs studying or working in fields of Civil Engineering, Architecture, Design and Mathematics.

As mathematicians with special interest in geometry, our idea was to pay homage to creating physical models for the study of ruled surfaces and to the geometry of line space, or more precisely its dimension.

A set of lines in three-dimensional space make a four-parameter set i.e. a four-dimensional set where ruled surfaces are its two-dimensional subsets.

These are the reasons why we decided to create models of the tesseract.



CAD-aided production process

- CAD modeling the object, the construction
- CAD modeling the pieces to be cut
- Laser-cutting
- Assembling and gluing
- Coloring and knitting

The Tesseract -- 4D-hypercube,

is a 4D-analogue to the cube,
a regular polyhedron -
its faces are regular polygons with the same number of sides,
the length of the sides being all equal,
with the same number of faces and edges at each vertex.

The 4D-hypercube can easily be imagined as
a cube inside a cube,
therefore consisting of 8 cubes –
a construction seemingly impossible in 3 dimensions

There are some ways to present a 4D-object
in a three-dimensional world.
For mathematicians it is usual to deal
with higher dimensional object and spaces,
but we rarely visualise such objects.

Common way to show
higher dimensional objects is by projections
into less-dimensional space
or using imagination.

How to comprehend, understand or visualise four dimensions?
Does the question invoke mathematics,
or can, should or must art also be included?

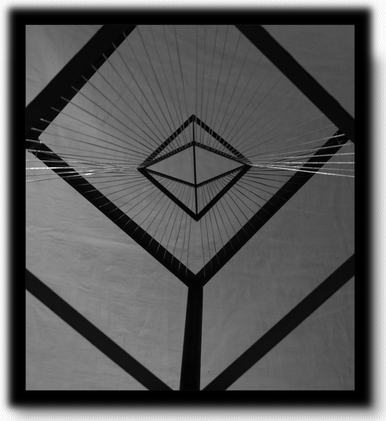
Mathematics, geometry is art –
shape, structure, form, relations, quantities,
measures, order, regularity, rules and exceptions.

Aesthetics – how to calculate the pleasurable,
how to measure the appealing – can we avoid mathematics?

Are mathematical constructions science or art?
Where is the borderline?

Art is craft – revival of the abstract.
From ideas to material objects.

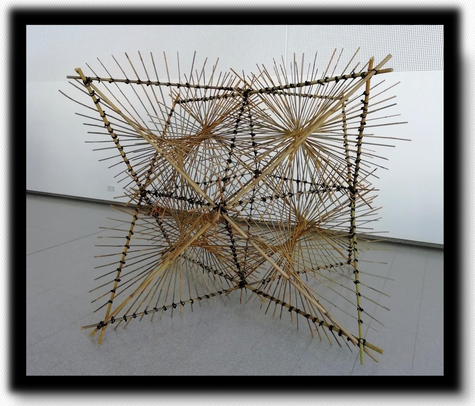
Art is an expression
of our knowledge, understanding and awareness.

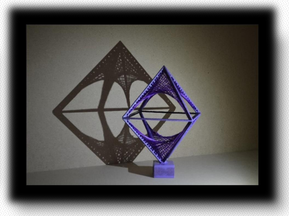


This portrayal of a tesseract also has the inner cube replaced by its Platonic dual – an octahedron, but the hyperbolic paraboloids connecting the cube with the octahedron are placed differently.
Each horizontal edge of the octahedron is connected with a skew edge of the cube by strings which geometrically define one system of rulings of a hyperbolic paraboloid.
The inspiration for this piece are again old string models, used and created for scientific and educational purposes of studying ruled surfaces in the 19th century.
Models of this kind can be found in the majority of technical universities throughout the world, and today, with the help of modern technologies such as CNC machines and CAD software, digital production of similar models can be and still is an excellent educational tool.

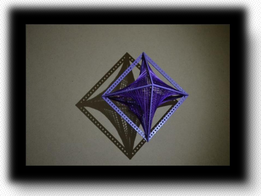
The frame of the tesseract is made from MDF panels cut by a laser cutter in the same way as the previous model, glued and colored.
One vertex of the frame is attached to a base suitable to withstand equilibrium of the model. The strings are recycled tread imitating metal.

This star-shaped bamboo sculpture is another representation of the tesseract.
The outer shape is an 8-star, stellated octahedron, named Stella octangula by Johannes Kepler in the early 17th century, whose 8 vertices form a cube with invisible edges.
We made it so, because a square, created only with bamboo sticks and rubber bands, is not a stable figure, while a triangle is. The cube is thus constructed as a convex hull of a regular compound of two tetrahedrons, self-dual Platonic solids with 4 vertices.
Geometrically, these two tetrahedrons are congruent and their intersection is an octahedron. Their combined Boolean difference consists of 8 smaller congruent tetrahedrons.
In each of the small tetrahedrons two hyperbolic paraboloids are placed to contain among them all the edges of the tetrahedron.
Small bamboo sticks present one system of rulings of each hyperbolic paraboloid and they are all parallel to the same vertical plane.





Dual to the Steiner construction of conics, by taking two intersecting lines we can construct tangents of a parabola, called a dual parabola (class curve). This simple construction is knitted inside 8 faces of an octahedron.



References

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