CROATIAN HEIGHT TRANSFORMATION MODEL

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ABSTRACT

After 1991, when the Republic of Croatia became independent, the “old” height reference system (HVRS1875) from the former Yugoslavia was still officially used. Soon the process of restoration and enhancement of geodetic datums and reference coordinate systems was initiated and in 2004, this resulted in the realisation of a new height reference system (HVRS71). In order to ensure the continuity of height data use along with an efficient height coordinates transformation between the two reference systems, the Croatian height transformation model (HTMV) was created. The model is based on height data of geometric levelling benchmarks of all levelling orders of accuracy, which are located on the Croatian territory, and which are included in both height reference systems. The HTMV was created on the basis of mutually independent modelling of datum and distortion components which are both comprised by the differences of the benchmarks' height coordinates. The model's datum component is a result of different height datums in the old (HVRS1875) and the new (HVRS71) height reference system. The referent height surfaces, realised by these datums, have different spatial orientation in respect to the Earth's body and are not mutually parallel. The model's distortion component is a result of different adjustment methods and procedures used to determine absolute benchmark heights and different geometric configurations of levelling lines and networks. Furthermore, it should be pointed out that levelling data from different time epochs were combined. The model's datum and distortion component were successfully integrated into a unique grid transformation model. The HTMV is a high accuracy transformation model, based on a homogenous density grid, meeting the criteria of efficiency, unambiguity and simplicity.

Keywords: height reference system, grid transformation, datum model, distortion model, the Republic of Croatia.

INTRODUCTION

After Croatia became independent, the process of innovating national geodetic datums and reference systems was initiated. The process was finalised in 2004 upon the decision of the Croatian government [1] by which the official geodetic datums and reference systems for Croatia were determined and have been officially in use since 1st January 2010. At that moment the new height datum and new height reference system were introduced, replacing the height reference system from the former Yugoslavia that was still used, which was based on the height reference system created during the Austro-Hungarian monarchy by the end of the 19th century. The introduction of the new height reference system and height datum has brought on, along with long-term benefits, some short-term difficulties. One of the most significant issues has been the
continuity of use of the already-existing height data, i.e., finding how to efficiently transform height coordinates (absolute heights) between the two reference systems. The Croatian height transformation model (HTMV) was created in order to solve that issue.

THE OLD AND THE NEW CROATIAN HEIGHT REFERENCE SYSTEM

During history three height reference systems have been used in Croatia [2]. Until recently, the height reference system officially used was the old Croatian height reference system for the epoch 1875 (HVRS1875), originally created during the Austro-Hungarian monarchy and which was significantly updated with measurement data of the so-called I levelling of high accuracy in the period from 1946 till 1963. Since the gravimetric measurements were not at disposal, a system of normal-orthometric heights was adopted. The height reference point of the height system was defined at the location of the tide gauge in Trieste (Italy) by calculating the mean level of the Adriatic sea from the results of continuous measurements of the sea level over a period of just one year (during 1875). The equipotential surface of Earth gravity field whose height position is defined by the absolute height of the starting (reference) benchmark HM1, determined in relation to the calculated mean level of the Adriatic sea, was adopted as the reference height surface. The network measurement data processing was gradual (combination of the Austrian precise levelling network and I levelling of high accuracy network), in accordance with the time dynamics of measurements, and the levelling network has never been adjusted as a whole, but in segments by conditional measurements adjustment and the least square method. This system was in use in Croatia until 1 January 2010, when the new reference system became valid.

The new reference system of the Republic of Croatia ("Croatian height reference system for the epoch 1971.5" - HVRS71) [3], [4], is based on the new height datum from the epoch 1971.5 and the new levelling network realised in the period from 1970 till 1973, i.e. the II levelling of high accuracy (IINVT). Since the gravimetric measurement was just partially realised, the normal-orthometric height system was kept. Unlike the old height datum, the reference points of the new height reference system were defined at locations of five tide gauges along the east coast of the Adriatic sea (Koper (Slovenia), Rovinj, Bakar, Split and Dubrovnik). The equipotential surface of Earth gravity field whose height position is defined at the locations of five tide gauges determined by the mean Adriatic sea level for the time epoch 1.7.1971, was adopted as the reference height surface. The mean sea level was calculated from the results of continuous measurements of the sea level over the period of 18.6 years. The height position of the reference height surface was defined by determining the absolute heights of referent benchmarks in the vicinity of the tide gauges. The levelling network was adjusted in its integral extent by indirect measurements adjustment and least square method [5].

The old (HVRS1875) and the new (HVRS71) Croatian height system were realised using different fundamental levelling networks with different course and methodologies of measurement data processing, spatially they are oriented with significantly different concepts of height datums realisations, but they encompass the same benchmark fields. Height coordinates of the same benchmarks in the old and the new height reference system are mutually comparable since the both systems are realised as normal-orthometric height systems. Furthermore, the differences between those coordinates are the basis for the creation of the mathematical height transformation model.
DATA BASIS FOR MODELLING

Due to the specific shape of the Republic of Croatia and in order to enhance connection between the north and the south part of the state, a certain number of available data for benchmarks on the territory of the neighbouring Bosnia and Herzegovina (BIH) was used. The positional distribution of benchmarks used for defining the height transformation model is shown on Figure 1.

The data basis consists of a set of 10564 benchmarks of geometric levelling of all levelling orders of accuracy that are encompassed by both Croatian height reference systems. Data for those benchmarks was successfully unified, homogenized and analyzed [6]. The data comprise: benchmark name, position in the ellipsoidal coordinate system – the Bessel ellipsoid (geodetic longitude $L$ and geodetic latitude $B$) and the differences of benchmarks’ height coordinates $\Delta H$ derivated from the expression:

$$\Delta H = H_{HVRS1875} - H_{HVRS71}, \quad (1)$$

where: $H_{HVRS1875}$ – is benchmarks’ height in the old height reference system HVRS1875 and $H_{HVRS71}$ – is benchmarks’ height in the new height reference system HVRS71.
From the homogenous set of spatial benchmark data (3D data) all global and local outliers have been identified and eliminated with various tests.

**MODELLING THE DIFFERENCES BETWEEN THE BENCHMARKS’ HEIGHTS**

The Croatian Height Transformation Model (HTMV) was created on the basis of separate modelling of two components which are both comprised in the differences between the benchmarks’ height coordinates $H_A$, (datum component $H_{AD}$ and distortion component $H_{Ad}$) and that were later integrated into a unique transformation model [6]. The model's datum component is a result of different height datums in the old and the new height reference systems. The referent height surfaces, realised by these datums, have different spatial orientation in respect of the Earth's body and are not mutually parallel. The model's distortion component is a result of different adjustment methods and procedures used to determine absolute benchmark heights and different geometric configurations of levelling lines and networks. Furthermore, it should be pointed out that levelling data from different time epochs were combined. The integrated mathematical model takes the form

$$\bar{H}_A = \bar{H}_{AD} + \bar{H}_{Ad},$$

where: $\bar{H}_A$ - is the modelled value of difference of the height coordinates between the old and the new system, $\bar{H}_{AD}$ - is the modelled value of the datum component and $\bar{H}_{Ad}$ - is the modelled value of the model’s distortion component. In accordance with the above, the transformation of absolute heights between the old and the new Croatian reference systems is defined by very simple mathematical expressions:

$$H_{HVRS71} = H_{HVRS1875} - \bar{H}_A,$$

$$H_{HVRS1875} = H_{HVRS71} + \bar{H}_A,$$

depending on the required direction of height data transformation. The data basis for modelling consists of a homogeneous set of differences of height coordinates $\Delta H$ of the benchmarks with known position in the ellipsoidal coordinate system $(L, B, H_A)$.

As the optimum solution for modelling of datum components of differences between the benchmarks’ height coordinates, the solution based on regression linear spatial parametric model defined with 7-parameter similar transformation has been adopted [7]. The mentioned model takes the form of regression function that for the empirical data of 10564 benchmarks, by applying the indirect adjustment and least squares method [8], results in the realization of datum’s transformation model specified with the expression

$$\bar{H}_{AD}(L, B) = 1986,140697a_{tx} + 479,4372746a_{ty} + 5899,999294a_{tz} - 0,01548713056a_{ex} + 0,06461378251a_{ey} - 0,000449605157a_m - 0,0004294240799a_f,$$

where: $a_{tx}, a_{ty}, a_{tz}$ - are coefficients of translation parameters, $a_{ex}, a_{ey}$ - are coefficients of rotation parameters, $a_m$ - is coefficient of the parameter of change in scale and $a_f$ - is
coefficient of the parameter of height datum change. The regression model provides, for any point with known ellipsoidal position \((L, B)\) along the model coverage area, the determination of the modelled value of datum’s component comprised in differences of height coordinates. The continuous datum model surface matching the expression (5) is shown in the Fig. 2.

Having defined the datum model, it is possible to reduce datum components \(\overline{H}_{\Delta D}\) determined by the model from the differences of the benchmarks’ height coordinates \(H_{\Delta}\), thus, the empirical values of the distortion components are determined

\[
H_{\Delta d} = H_{\Delta} - \overline{H}_{\Delta D}.
\]  

Based on those distortion components empirical values, for all of 10564 benchmarks \((L, B)\), the set of spatial data exists for the creation of the distortion model. As an optimum solution for the distortion model creation, the spatial interpolation modelling method of the “Minimal Curvature Surface - MCS” has been adopted [6], [9]. The distortion model surface has been presented on Figure 3.
CROATIAN HEIGHT TRANSFORMATION MODEL – VERSION HTMV08-v.1

In order to connect the datum and distortion model into an integral transformation model, the regression datum model was translated into the grid form (datum model) with identical grid parameters as were used at creation of distortion model. At the creation of the distortion model (application of MCS), the model coverage area was covered with a regular square network of lines (grid) of known planar position. By modelling distortion components associated to benchmarks and according to their known ellipsoidal position \((L, B)\) the modelled distortion components associated to grid nods’ (points on intersection of grid lines) were determined. It should be pointed out that, in the case of grid model application, the modelled value of datum and distortion component determination for any point of known planar position \((L_P, B_P)\), within the model coverage area, is reduced to the application of bilinear interpolation (Fig. 4). Since the datum model has been transformed into a form of grid model with the same dimensions and characteristics of the grid that was used when creating the distortion model, their very simple integration in the HTMV is made possible and very simple. The adopted grid cells dimensions are \(\Delta L = 45''\) and \(\Delta B = 30''\) (approx. 1 km x 1 km).
Figure 4. Grid cells, transformation parameters and position of point P

Spatial surface of the Croatian Height Transformation Model, created by merging of the datum and distortion model, on the total model coverage area is presented in Fig. 5.

Figure 5. Surface of Croatian height transformation model - HTMV
The model was realised with an adequate grid file containing, as transformation parameters, the data of modelled values of absolute height differences between height systems on grid nodes. The HTMV provides an unambiguous and simple transformation of height coordinates of points with known position between Croatian height reference systems. Indicators of models’ quality were determined from the deviation $\varepsilon$ between original $H_\Delta$ and modelled $\overline{H}_\Delta$ differences of the benchmarks’ height coordinates.

$$\varepsilon = \overline{H}_\Delta - H_\Delta.$$  

(7)

The internal model’s accuracy expressed with standard deviation is 2.1 mm ($\varepsilon$ for all of the 10564 benchmarks), and external model’s accuracy ($\varepsilon$ for the 1613 benchmarks - completely independent data control set) is 8.2 mm. Internal and external accuracy do not show the accuracy of final transformed heights determined by transformation, but show the accuracy of transformation process, in terms of conservation of original accuracy of height data that are transformed with the model. Both standard deviations indicate quite high accuracy of transformation. It can be concluded that the HTMV is suitable for the transformation of height coordinates of points with known positions on the Croatian territory and that it is really a high-accuracy transformation model.

REFERENCES


