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# **Nove Geometrijske Metode Geodezije u Svemiru**

**Pioneering the Geometrical Approach of Space Geodesy  
and Paving the Way Towards Planetary Geodesy**

*Geometrijske metode satelitske geodezije  
i trasiranje puta prema planetarnoj geodeziji*

**Dr.-Ing. Dražen Švehla**

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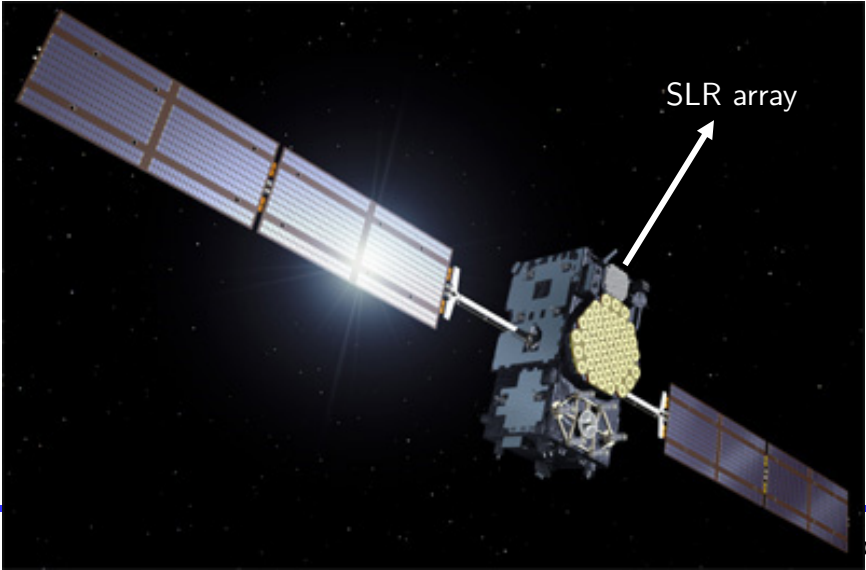
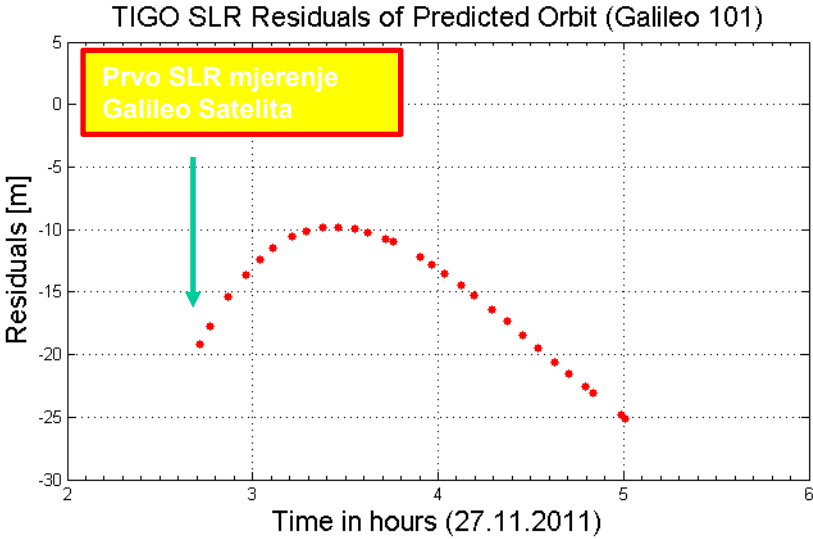
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- Metoda faznih satova i određivanje ambiguiteta (zero-differences)
- Određivanje terestričkog referentnog sustava kombiniranjem GPS, SLR i DORIS mjerenja sa satelita
- Prva SLR bazisna linija sub-milimetarske točnosti
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- Galileo H-maser: određivanje  $J_2$  koeficijenta gravitacijskog polja Zemlje
- Galileo H-maser: prvo geometrijsko mjerenje termalne inercije iluminiranog Galileo satelita
- Geometrijska metoda računanja sfernih harmonika, vremenskih promjena sile teže i orbita satelita

# Prva SLR Mjerenja Galileo Satelita

## ESA Press Release Dec/2011



esa galileo iov European Space Agency

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Galileo IOV at a glance

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- Objectives
- About Galileo
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  - Soyuz launch site
- After launch
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24-Oct-2012

Galileo IOV launch website

News

First laser measurements of Europe's Galileo satellites made from Chile

15 December 2011  
The first laser ranging of Europe's new Galileo navigation satellites has been achieved from Concepción in Chile. Laser contact with the satellites at an altitude of 23 230 km has provided distance measurements with subcentimetre accuracy.

The Transportable Integrated Geodetic Observatory, TIGO, in Concepción, performed the world's first laser ranging to the first Galileo satellite on 27 November at 02:45 GMT, and to the second satellite two days later at 10:05 GMT, using a near-infrared laser beam.

Laser ranging

Observations: in 2006 its radio moon mission, SMART-1, during lunar surface.

Galileo in tune: first navigation signal transmitted to Earth

Federal agency for Cartography and Geodesy (BKG)

International Laser Ranging Service

Contact for information

International Laser Ranging Service

TIGO is owned by the German Bundesamt und Geodäsie. It has been operated by the Universidad (UdC) and Instituto Geográfico (IGM) since 2004. TIGO has established various types of geodetic monuments.

TIGO was the 40-station Laser Ranging Service network to range the Galileo satellites. The Galileo with Herstmonceux in the UK and Matera in Italy. Satellite Laser Ranging stations to succeed.

As well as being widely used for precise orbit determination of satellites, laser ranging is also employed for calibrating satellite instruments, contributing to the International Terrestrial Reference Frame (Earth's standardised geodetic coordinate system) and

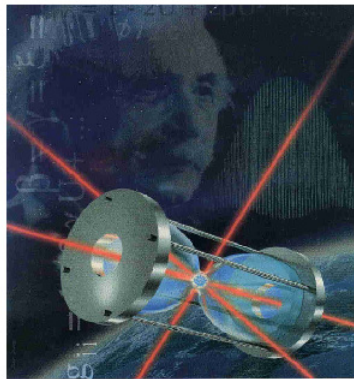


Topical Team on Geodesy  
Applications of the ACES Mission

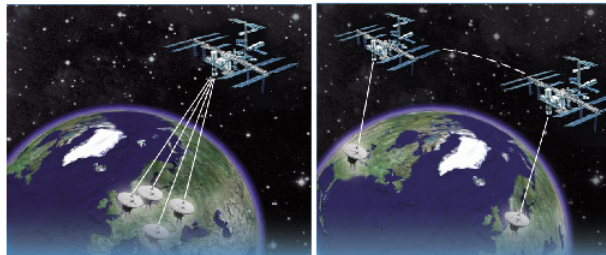
# ACES and FUTURE GNSS-BASED EARTH OBSERVATION and NAVIGATION

26 – 27 May 2008

Institute of Astronomical and Physical Geodesy  
Technische Universität München



(Credit CNES)



(Credit ASTRIUM)



Topical Team on Geodesy  
Applications of the ACES Mission

## ACES and FUTURE GNSS-BASED EARTH OBSERVATION and NAVIGATION

26 – 27 May 2008, Munich, Germany

Institute of Astronomical and Physical Geodesy  
Technische Universität München, Germany

## PROCEEDINGS



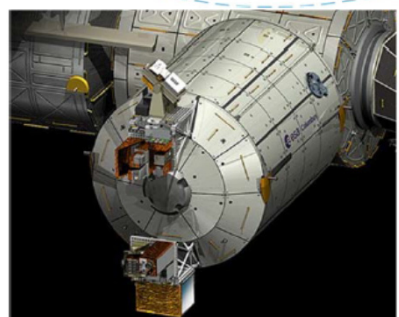
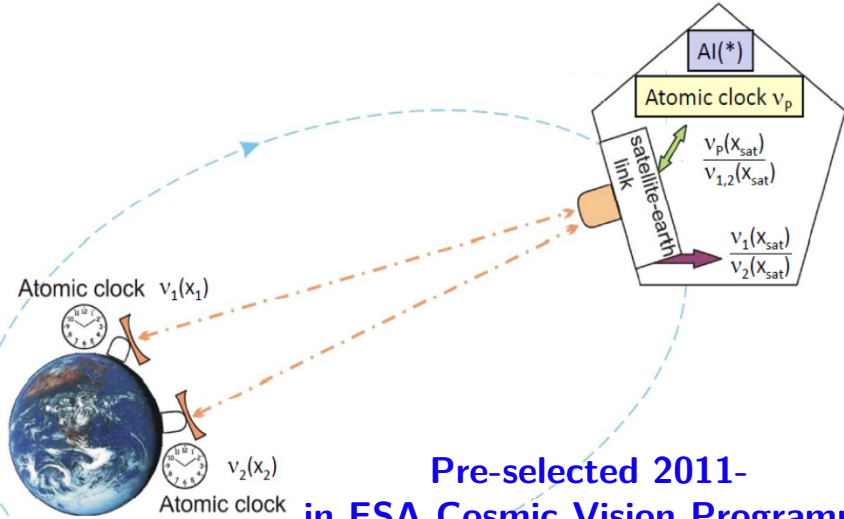
# Koordinator Satelitske Misije - Geodetski Dio

## Space-Time Explorer (STE-QUEST)

ESA Cosmic Vision (M-Class Mission)

**Terestrički/Nebeski Koordinatni Sustav**  
**Secondary Objective of the Mission**

Pre-selected 2011-  
in ESA Cosmic Vision Programme



**Galileo@ISS**  
**now GEROS**  
Selected 2012



**ACES**  
Geodesy Part Selected 2008

**OPS-SAT**  
Selected 2012



**Galileo@ISS or GEROS**

**GNSS Altimetry** (coherent GNSS reflectometry)  
TU München, GFZ, DLR, 25 co-authors

**Standalone geodesy mission on ISS**

Proposal (100 pages) submitted in 2008 ESA Topical Team on Geodesy

**ACES – Geodesy Part**

**Time/Frequency Transfer**, GNSS Radio-Occultation,  
(TU München, GFZ, DLR)

**OPS-SAT (CubeSat)**  
**SLR Demonstrator** (Graz)

# GNSS-Reflectometry (altimetrija) na Svemirskoj Postaji

Proposal Number: AO-2004-143

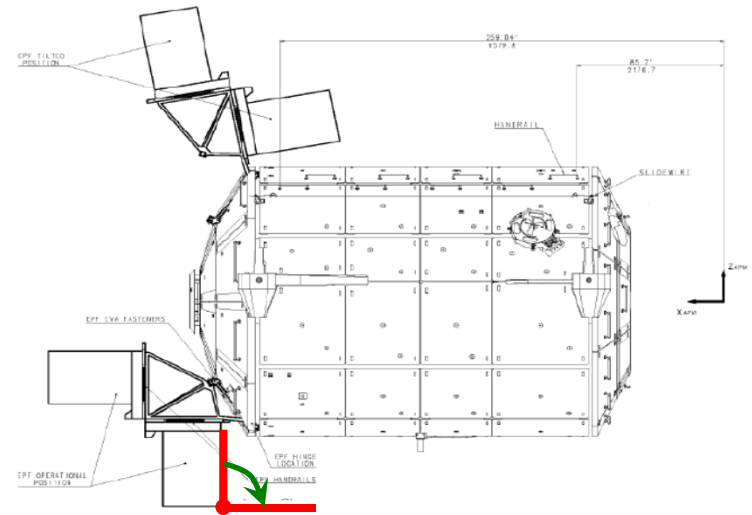
TUM IAPG

## Galileo Onboard the International Space Station GALILEO-ISS

Coordinator: D. Svehla



München, 2008

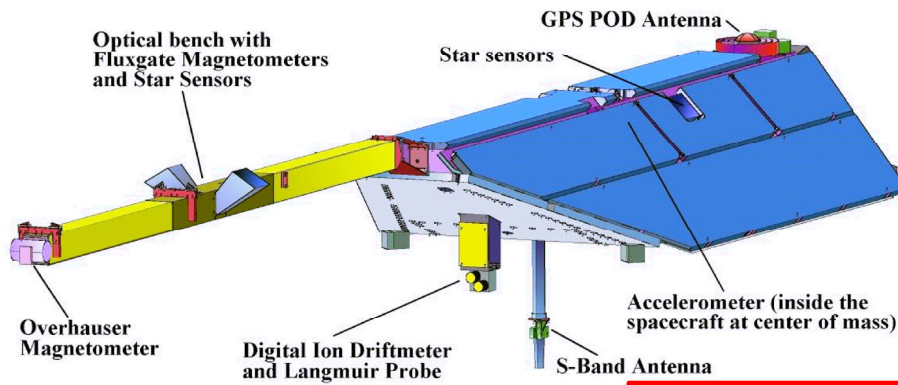


- ESA Phase A/B Studije 2013-

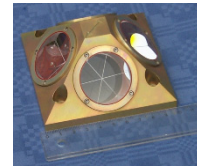
# GPS na Satelitu u LEO Orbiti

## CHAMP satelit

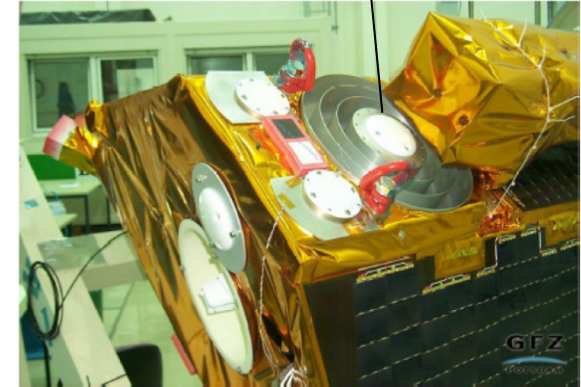
visina 400 km



Laser retro-reflektor



Choke-ring  
GPS antena



## GOCE satelit

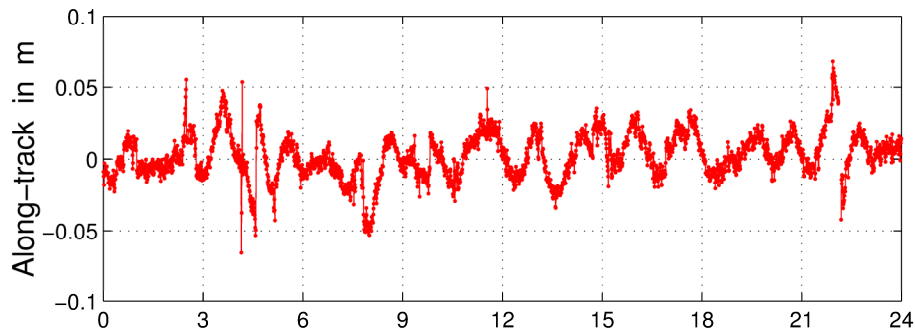
visina 220 km



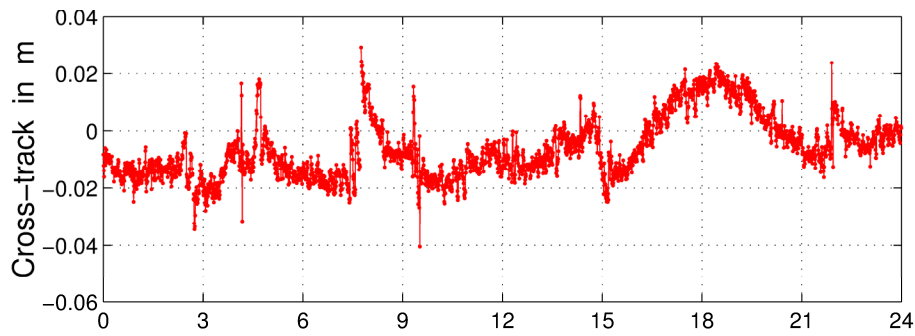
# Prva Geometrijska Trajektorija Satelita

## CHAMP

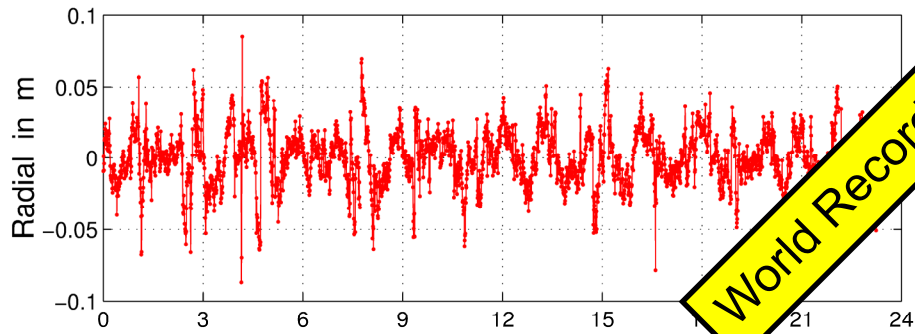
RMS=1.6 cm



RMS=1.2 cm



RMS=1.9 cm

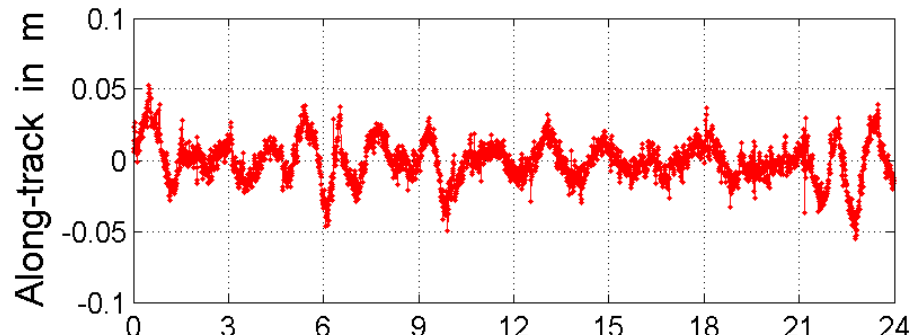


World Record

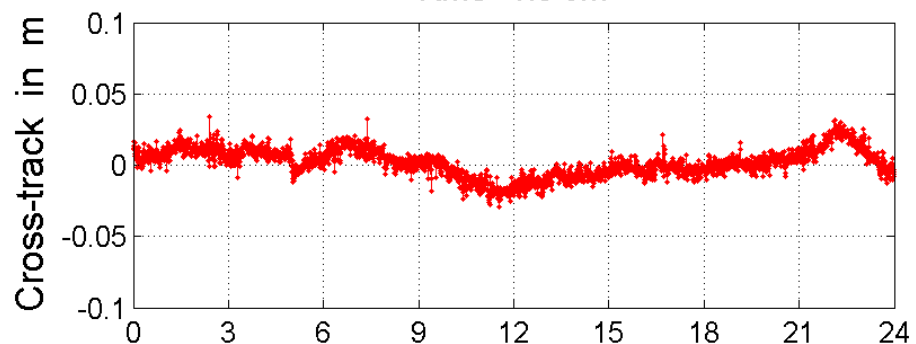
Day 200/2002

## GRACE-A

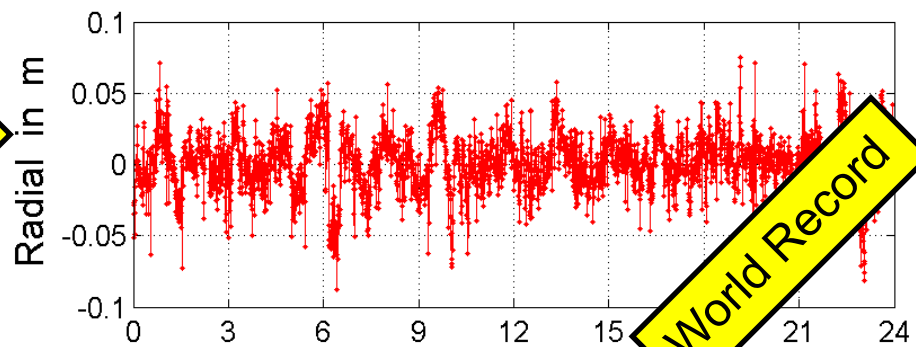
RMS=1.4 cm



RMS=1.0 cm



RMS=2.1 cm

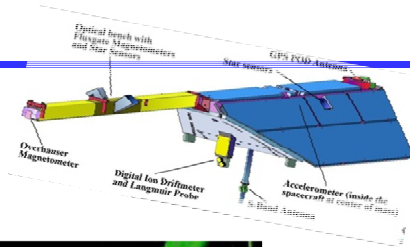


World Record

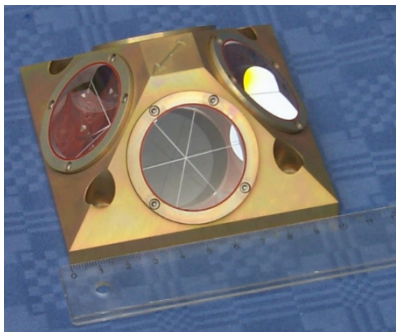
Day 200/2003



# Satelitska Laserska Mjerenja (SLR)



Laser retro-reflektor

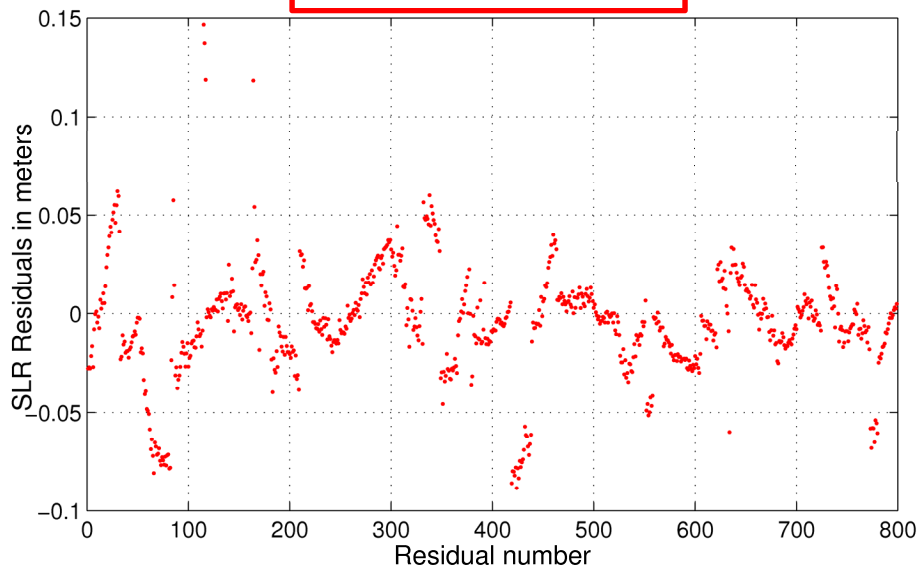


DINAMIČKA orbita

KINEMATIČKA orbita

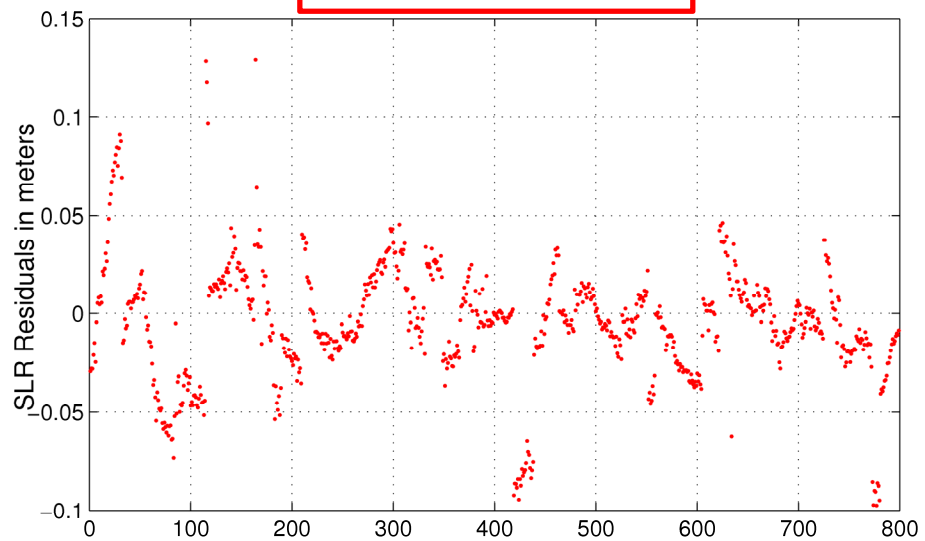
SLR residuals: CHAMP reduced-dynamic orbit, GPS week 1175/2002

Standard Deviation = 23 mm



SLR residuals: CHAMP kinematic orbit, GPS week 1175/2002

Standard Deviation = 25 mm



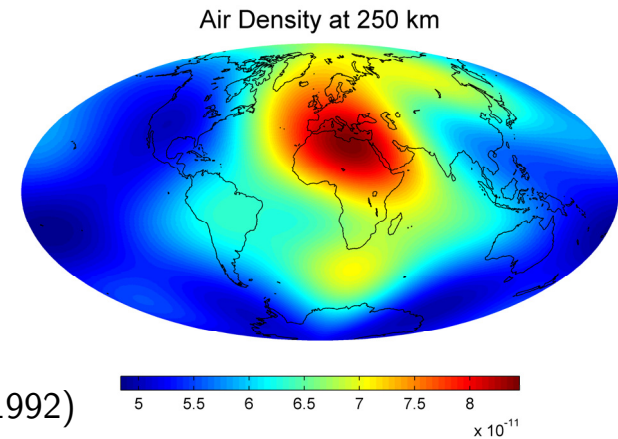
# Četiri Fundamentalne Metode Određivanja Orbita Satelita

1) Dinamička metoda (e.g. Topex-Poseidon, Tapley et al. 1992)  
numerička integracija

2) Reducirana-dinamička metoda (Topex-Poseidon, Yunck et al. 1992)  
numerička integracija

3) Kinematička metoda (CHAMP, Švehla and Rothacher 2002)  
Geometričke orbite bazirane na Faznim Parametrima Satova

4) Reducirana-kinematička metoda (CHAMP, Švehla and Rothacher 2003)  
Geometričke orbite sa dinamičkom informacijom



# Gravitacijsko Polje Zemlje Bazirano na Geometrijskoj Orbiti CHAMP Satelita

Gravitacijski  
potencial ←

$$V = \frac{1}{2} \left( \frac{d\vec{r}}{dt} \right)^2 - \int_{\vec{r}} \vec{a}_{non-gravitational} d\vec{r} - \int_{\vec{r}} \vec{a}_{tides} d\vec{r} + C$$

kinematička orbita  
(GPS trajektorija)

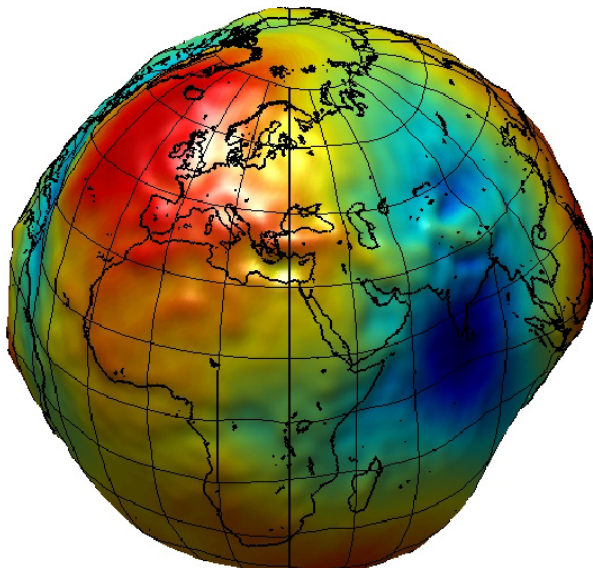
akcelerometer

plimni valovi

**CHAMP/GRACE/GOCE kinematičke orbite korištene po čitavom svijetu!!!**

## EGM2008 Gravity Model

je baziran na GRACE kinematičkoj orbiti



Austria	(gravity)
Canada	(gravity)
China	(gravity/POD)
Denmark	(gravity)
Germany	(gravity/POD)
Japan	(POD)
Netherlands	(gravity/POD)
Switzerland	(POD)
Taiwan	(gravity/POD)
UK	(gravity)
US	(POD)

# GOCE Gravitacijski Modeli - Kinematičke Orbite

Celestial Mechanics Approach

AIUB – University of Bern

Short-Arc Approach

TU Graz

Averaged Acceleration Approach

DEOS Delft, TU Delft, University of Louxemburg

Point-wise Acceleration Approach

University of Stuttgart/Austrian Academy of Sciences

Energy Balance Approach

TU Graz, TU München

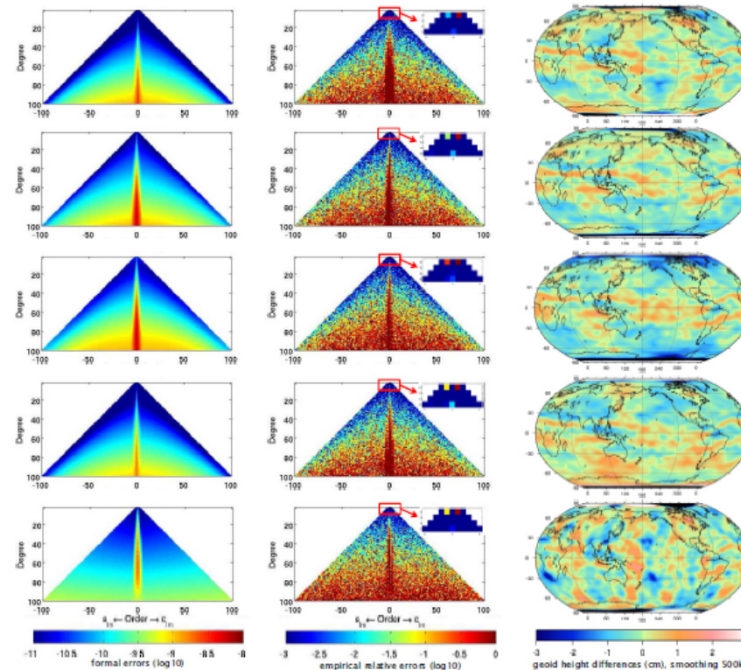
Celestial Mechanics Approach

Short-Arc Approach

Averaged Acceleration Approach

Point-wise Acceleration Approach

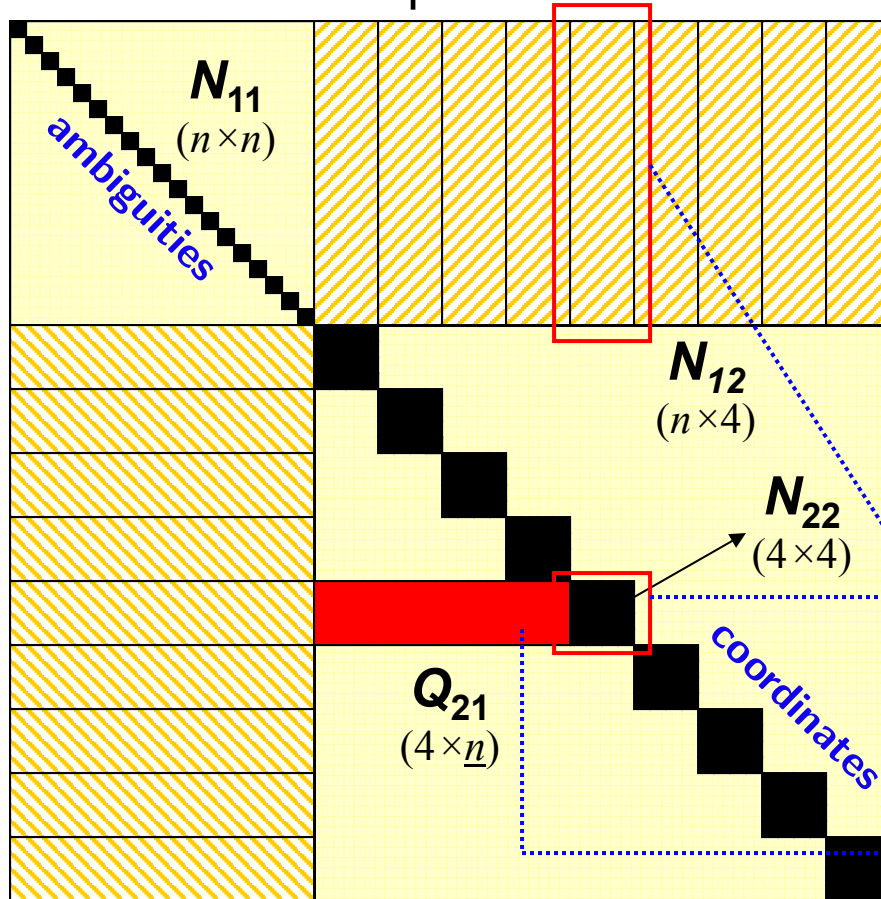
Energy Balance Approach



(Baur et al, 2013)

# Matrica Normalnih Jednadžbi

Normal equation matrix



$$\begin{bmatrix} N_{11} & N_{12} \\ N_{21} & N_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

$x_1$  = ambiguities

$x_2$  = epoch-wise kinematic coordinates & clocks

$$(N_{11} - N_{12}N_{22}^{-1}N_{21})x_1 = b_1 - N_{12}N_{22}^{-1}b_2$$

back substitution:

$$x_2 = N_{22}^{-1}(b_2 - N_{21})x_1$$

$$1) \quad Q_{22} = N_{22}^{-1} + N_{22}^{-1}N_{21} Q_{11} N_{12}N_{22}^{-1}$$

( $4 \times 4$ )      ( $4 \times 4$ )      ( $n \times n$ )

$$2) \quad Q_{12} = -Q_{11}N_{12}N_{22}^{-1}$$

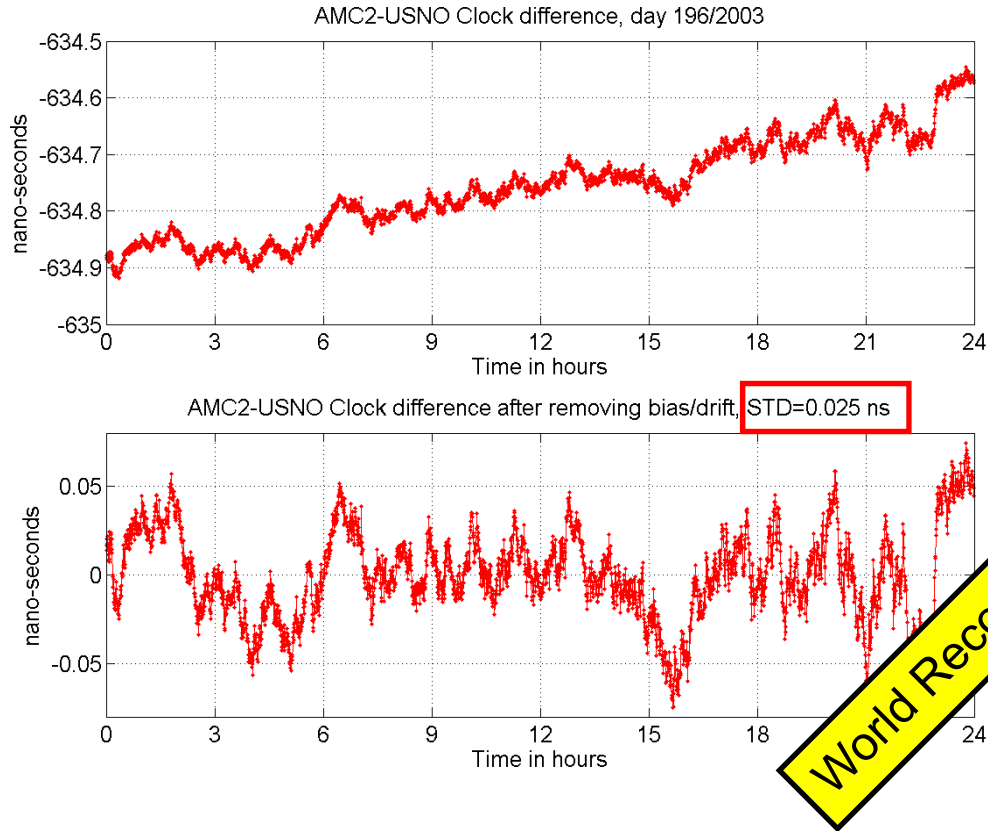
( $n \times 4$ )

$$3) \quad Q_{21} = -N_{22}^{-1}N_{12}N_{22}^{-1}$$

( $4 \times n$ )      ( $4 \times 4$ )

# Inter-Kontinentalna Usporedba Frekvencije Pomoću GPS-a Fazni Parametri Satova Satelita

## Colorado Springs (AMC2) – USNO (US Naval Observatory)



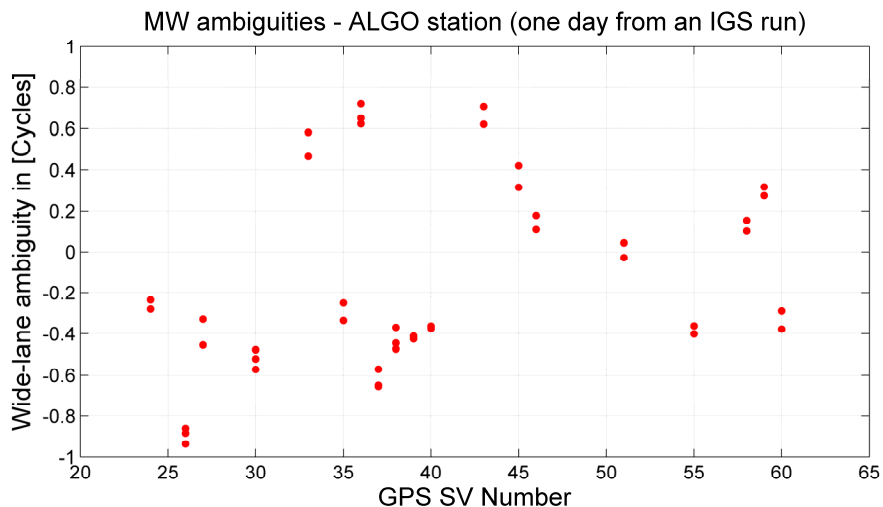
25 ps  $\approx$  7 mm

Stabilnost GPS prijavnika i H-maser-a

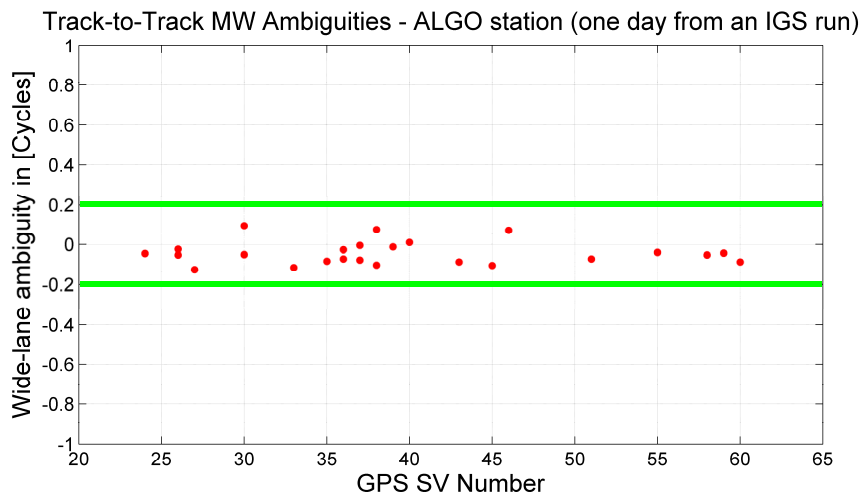
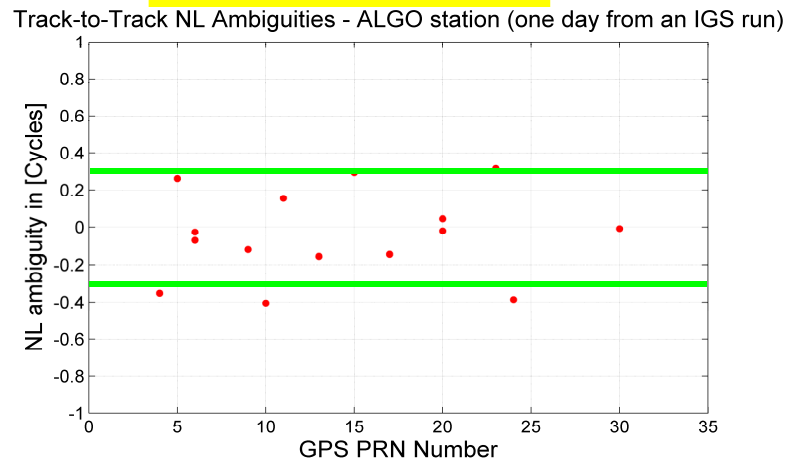
# Određivanje Ambiguiteta Pomoću Faznih Parametara Satova

## Track-to-Track Ambiguiteti

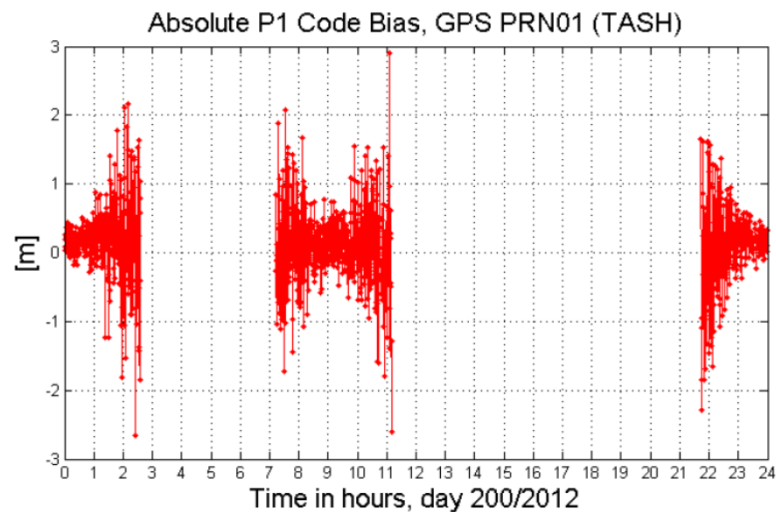
### Wid-Lane Ambiguity



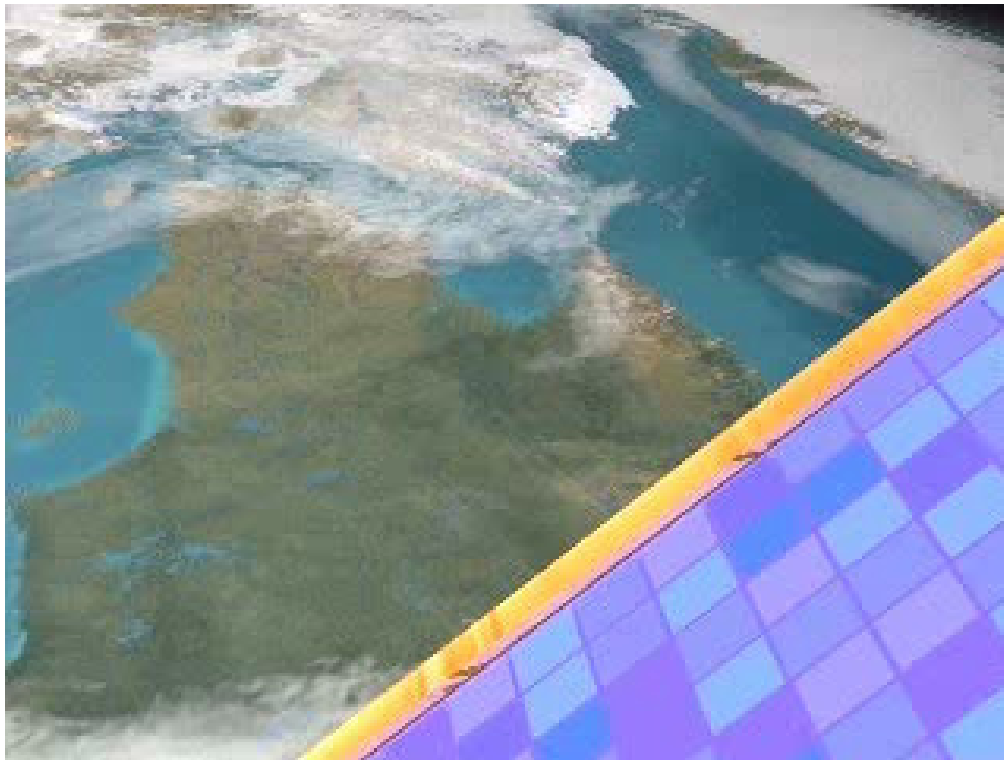
### Narrow-Lane Ambiguity



## Absolutna Kalibracija Kodnih Mjerenja

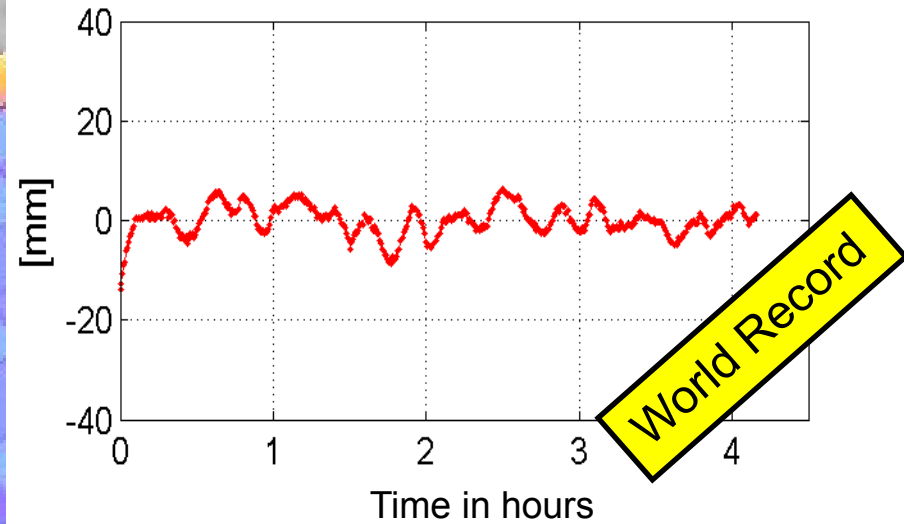


# Prva GPS Bazisna Linija u Svemiru



Usporedba sa Faznim Mjerenjima  
između satelita (K-Band)

**RMS= 2.8 mm (0.7 mm)**



(Status 2003-2004)



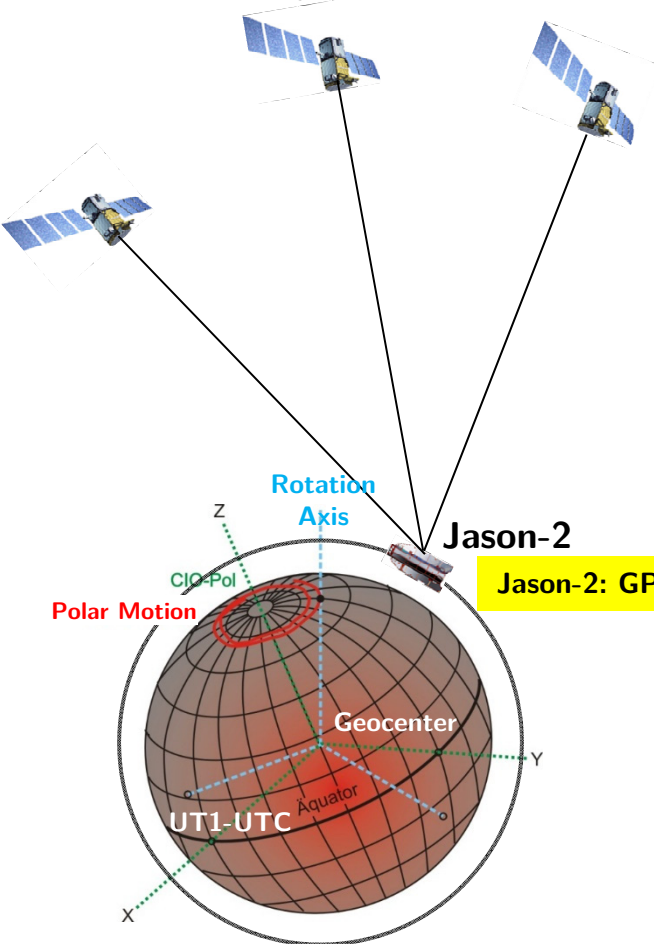
# Terestrički Referentni Sustav Baziran na 30 GPS Satelita + Jason-2 (GPS, SLR, DORIS)

Konstelacija 30 GPS Satelita

## Kordinate Geocentra

### Weekly Geocenter Coordinates

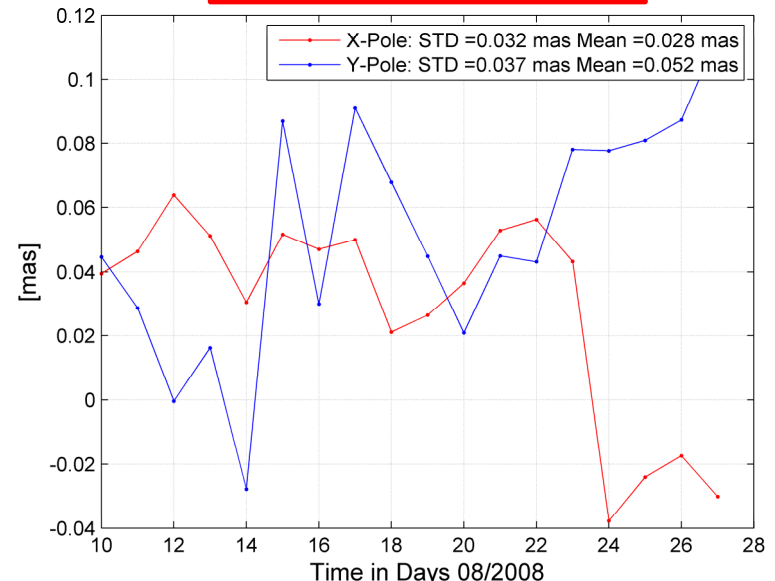
Week 1	Week 2	Week 3
$dx = -0.83 \text{ mm}$	$dx = -1.78 \text{ mm}$	$dx = -1.72 \text{ mm}$
$dy = -0.94 \text{ mm}$	$dy = -1.67 \text{ mm}$	$dy = -1.22 \text{ mm}$
$dz = -5.90 \text{ mm}$	$dz = -5.75 \text{ mm}$	$dz = -5.60 \text{ mm}$
$rx = 0.021 \text{ mas}$	$rx = 0.067 \text{ mas}$	$rx = 0.059 \text{ mas}$
$ry = 0.052 \text{ mas}$	$ry = 0.055 \text{ mas}$	$ry = -0.011 \text{ mas}$
$rz = -0.051 \text{ mas}$	$rz = -0.077 \text{ mas}$	$rz = -0.051 \text{ mas}$
scale = 0.13 ppb	scale = 0.14 ppb	scale = 0.16 ppb



Jason-2: GPS+SLR+DORIS

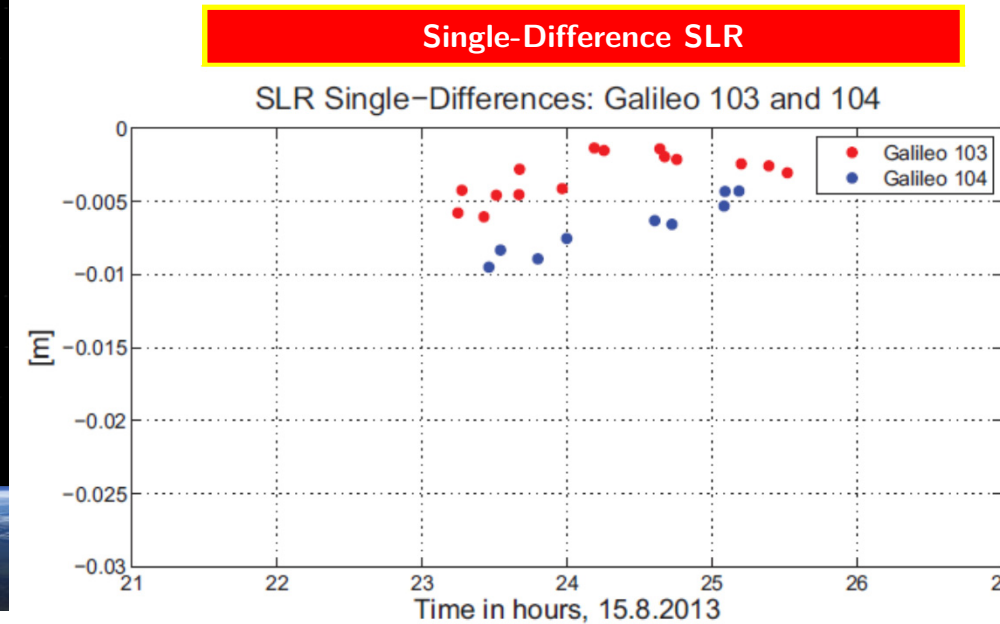
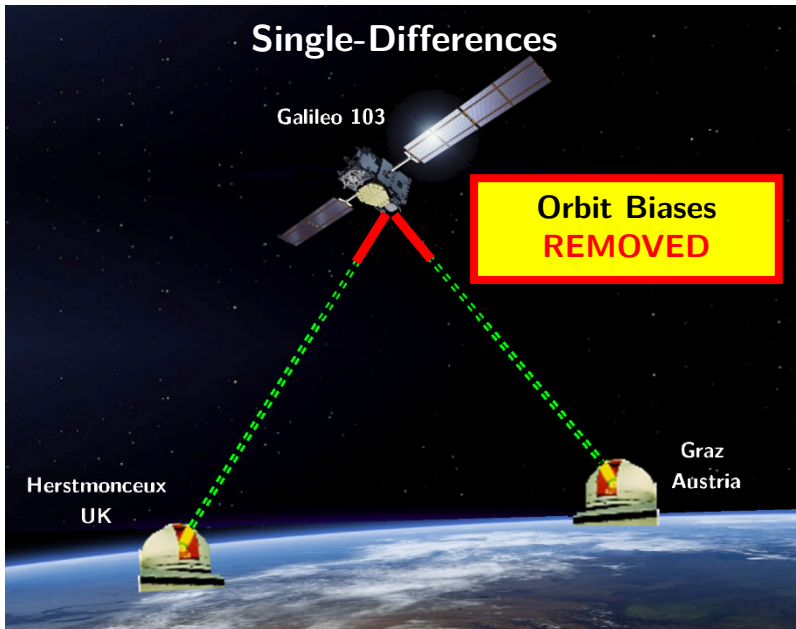
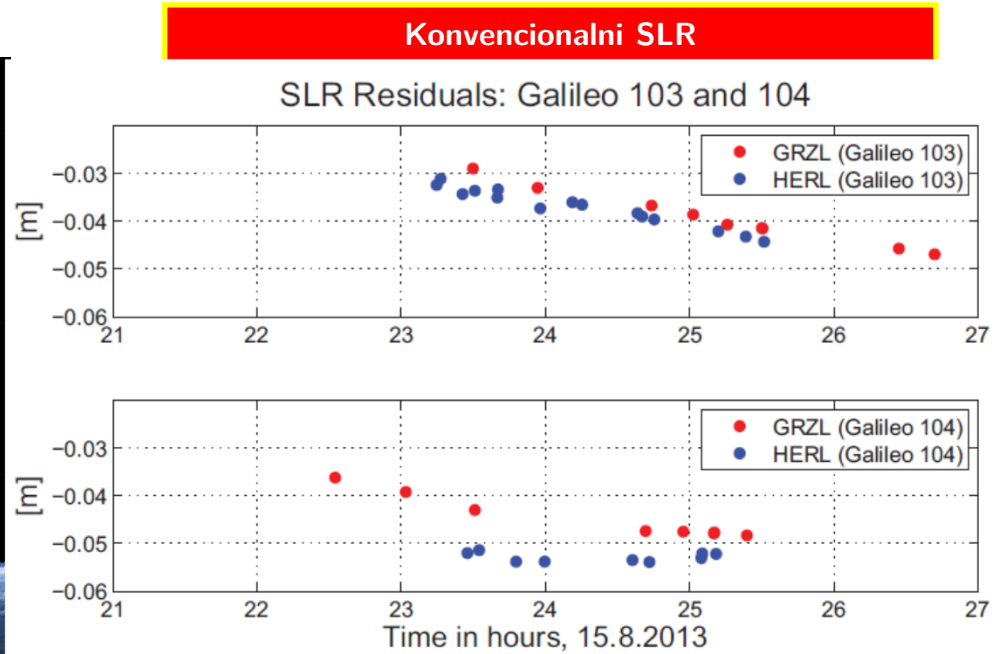
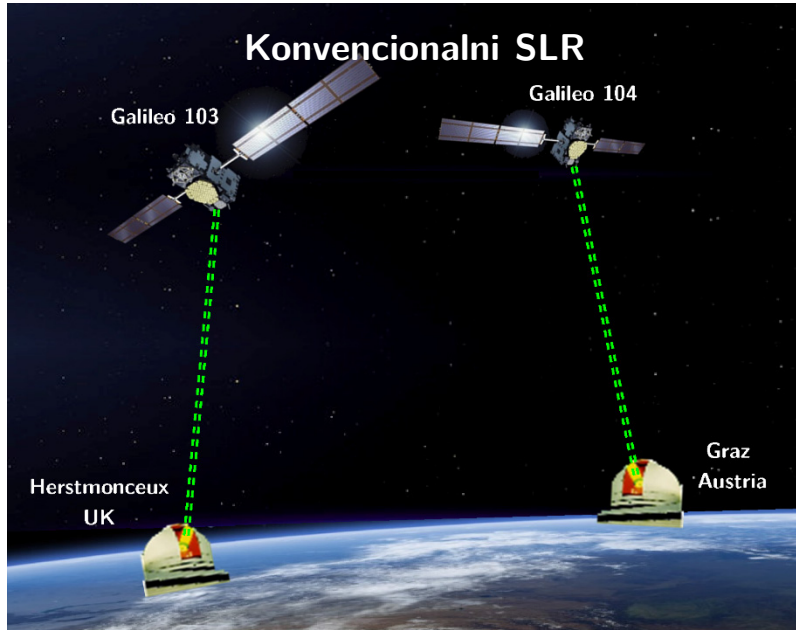
Polar Motion

## Polar Motion



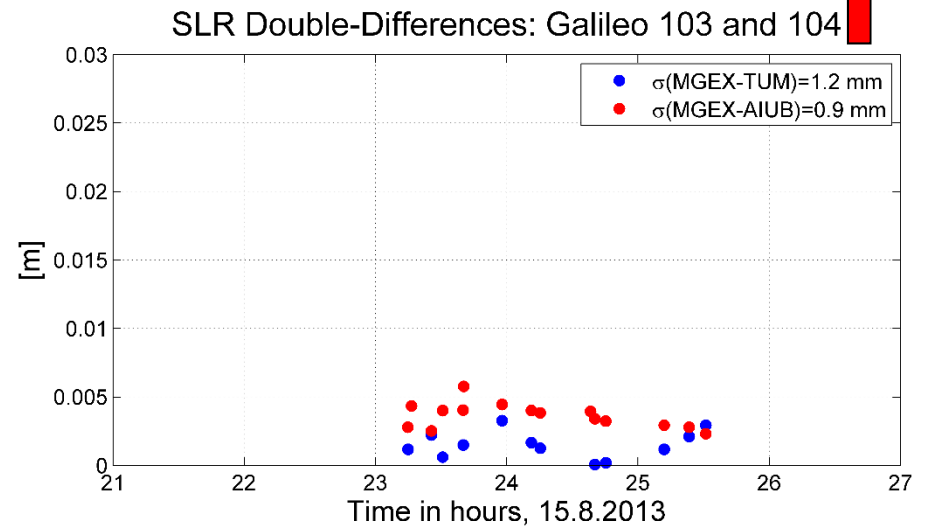
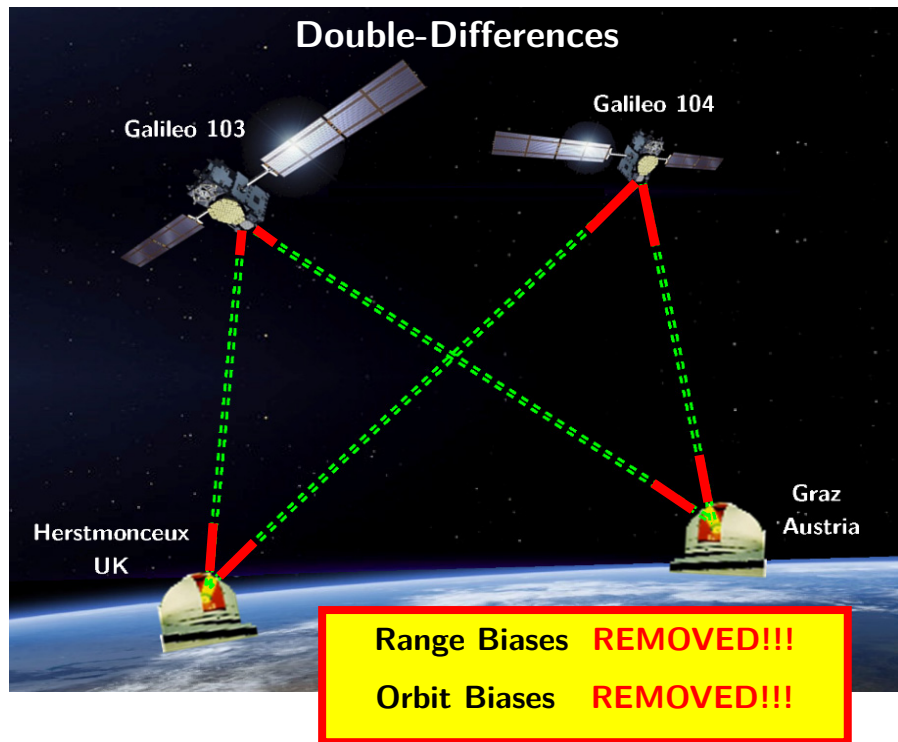
**Global Geodetic Observing System (GGOS):**  
**CILJ: 1 mm točnost, 0.1 mm/yr stabilnost**

# Prva SLR Bazisna Linija



# Prva SLR Bazisna Linija

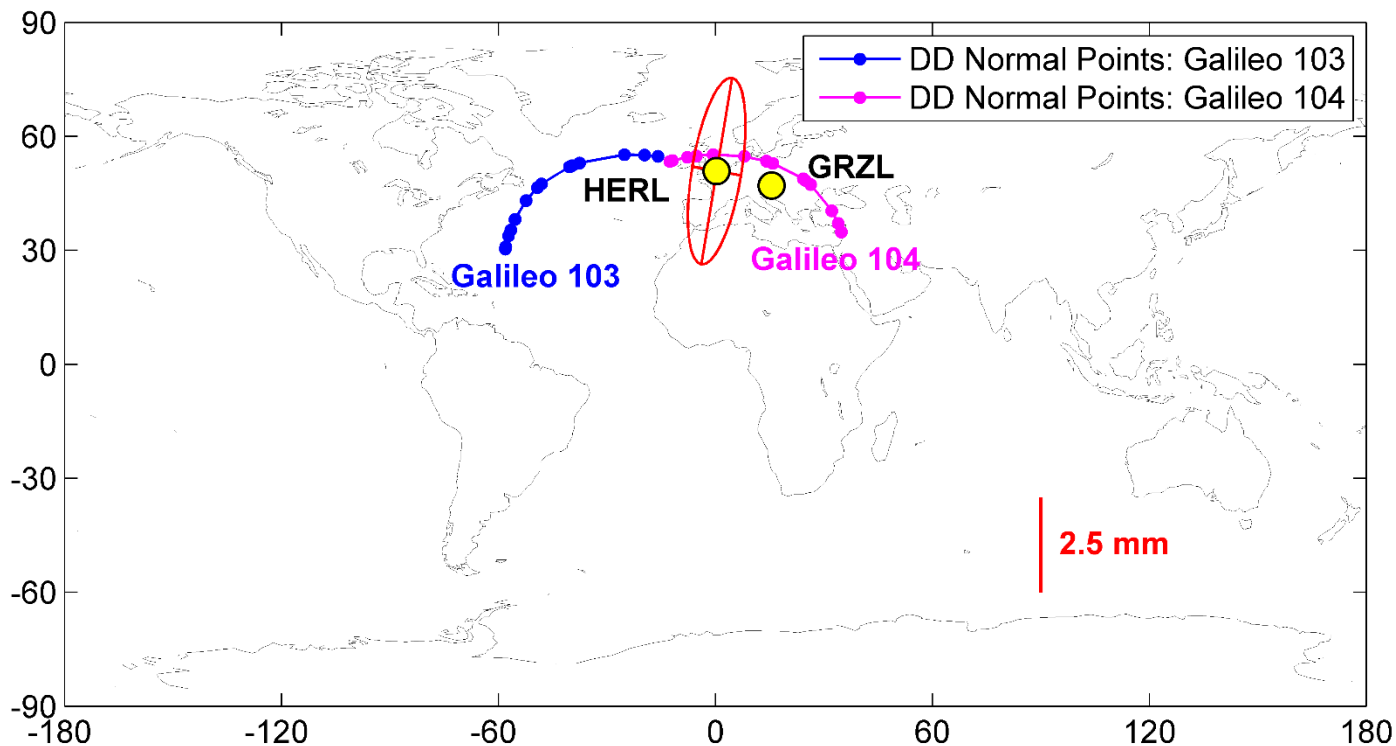
**RMS = 0.5 - 0.6 mm**



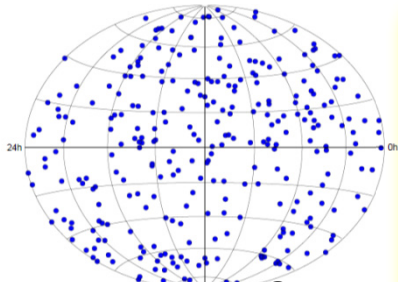
# Prva SLR Bazisna Linija: Elipsa Pogreška

Estimated coordinates of HERL in (mm) using SLR double-differences from GRZL	
MGEX-TUM Orbits $\sigma_0 = \pm 1.2$ mm	MGEX-AIUB Orbits $\sigma_0 = \pm 0.7$ mm
N=1.2±2.5 E=2.1±0.8	N=4.2±7.2 E=8.1±2.4

First SLR Double-Difference Baseline



# Terrestrial and Celestial Reference Frame




Extragalactic Sources



Quasar

Geophysical Research Abstracts  
 Vol. 16, EGU2014-7934-2, 2014  
 EGU General Assembly 2014  
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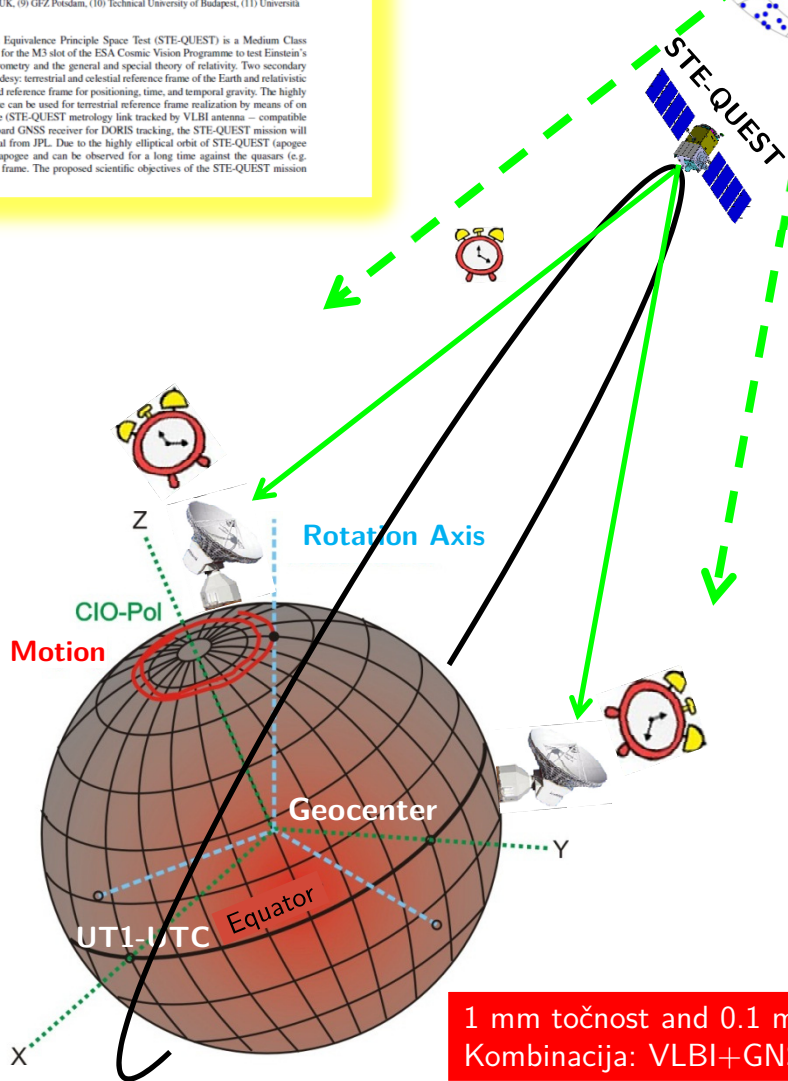
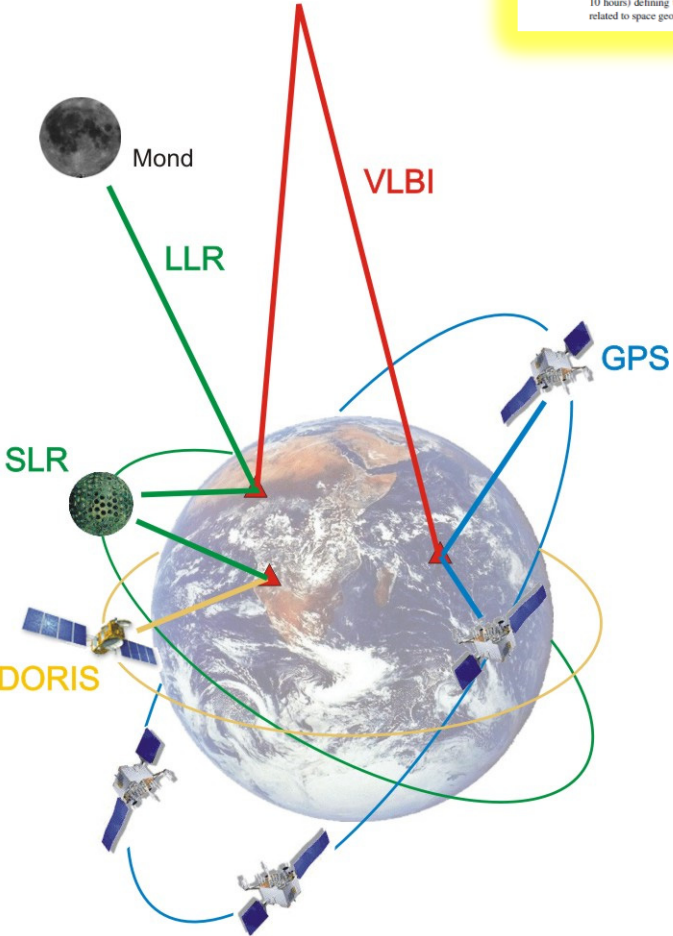
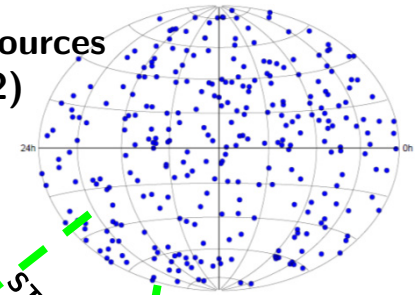
**Terrestrial and Celestial Reference Frame Realization with Highly Elliptical Orbit - The ESA STE-QUEST Mission**

Drazen Svehla (1), Markus Rothacher (2), Urs Hugentobler (3), Axel Nothnagel (4), Pascal Willis (5), Richard Biancale (6), Marek Ziebart (7), Graham Appleby (8), Harald Schuh (9), József Ádám (10), Luciano Iess (11), and Luigi Cacciapuoti (12)

(1) on the way to ESA/ESTEC, (2) ETH Zurich, (3) TU München, (4) University of Bonn, (5) Institut Physique du Globe de Paris, (6) CNRS, (7) UCL London, (8) NERC UK, (9) GFZ Potsdam, (10) Technical University of Budapest, (11) Università La Sapienza Rome, (12) ESA/ESTEC

The Space-Time Explorer and Quantum Equivalence Principle Space Test (STE-QUEST) is a Medium Class fundamental physics mission pre-selected for the M3 slot of the ESA Cosmic Vision Programme to test Einstein's Equivalence Principle using atom interferometry and the general and special theory of relativity. Two secondary mission objectives are related to space geodesy: terrestrial and celestial reference frame of the Earth and relativistic geodesy aiming at the realization of unified reference frame for positioning, time, and temporal gravity. The highly elliptical orbit of the STE-QUEST satellite can be used for terrestrial reference frame realization by means of on board GNSS, SLR and VLBI radio source (STE-QUEST metrology link tracked by VLBI antenna – compatible with VLBI2010). By upgrading the on board GNSS receiver for DORIS tracking, the STE-QUEST mission will be similar to the GRASP mission proposal from JPL. Due to the highly elliptical orbit of STE-QUEST (apogee <math>< 50\,000\text{ km}</math>) the satellite dwells in the apogee and can be observed for a long time against the quasars (e.g. 10 hours) defining the celestial reference frame. The proposed scientific objectives of the STE-QUEST mission related to space geodesy are as follows:

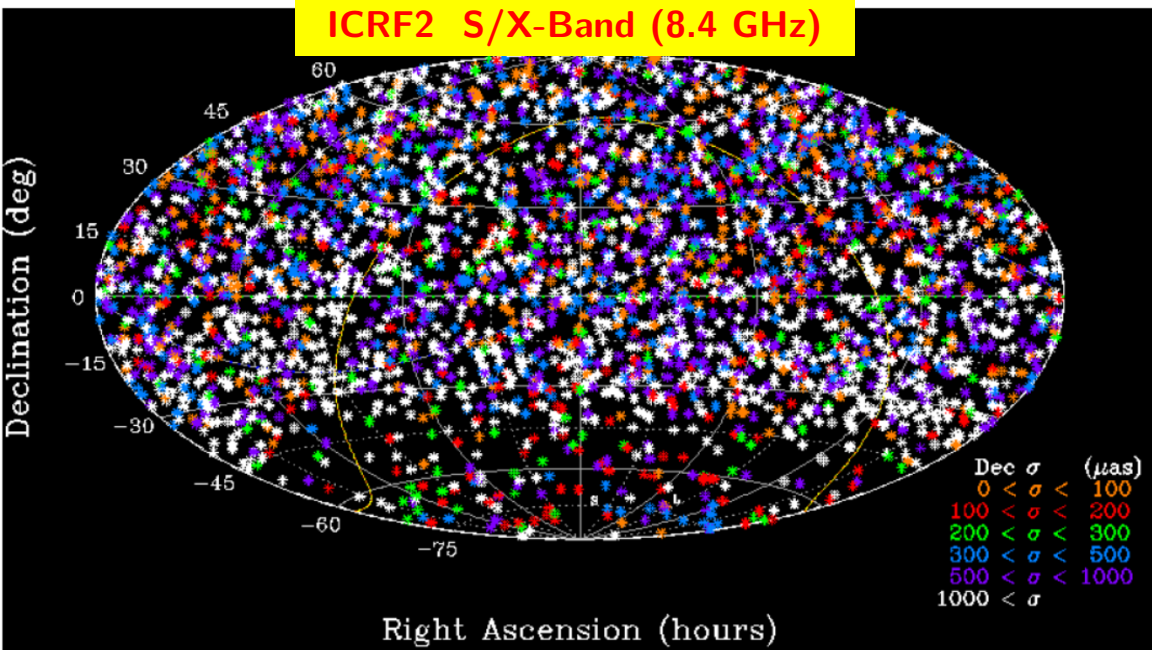
## Extragalactic Sources Kvazars (ICRF2)



1 mm *točnost* and 0.1 mm/yr *stabilnost*  
 Kombinacija: VLBI+GNSS+SLR+DORIS

# Nebeski Referentni Koordinatni Sustav

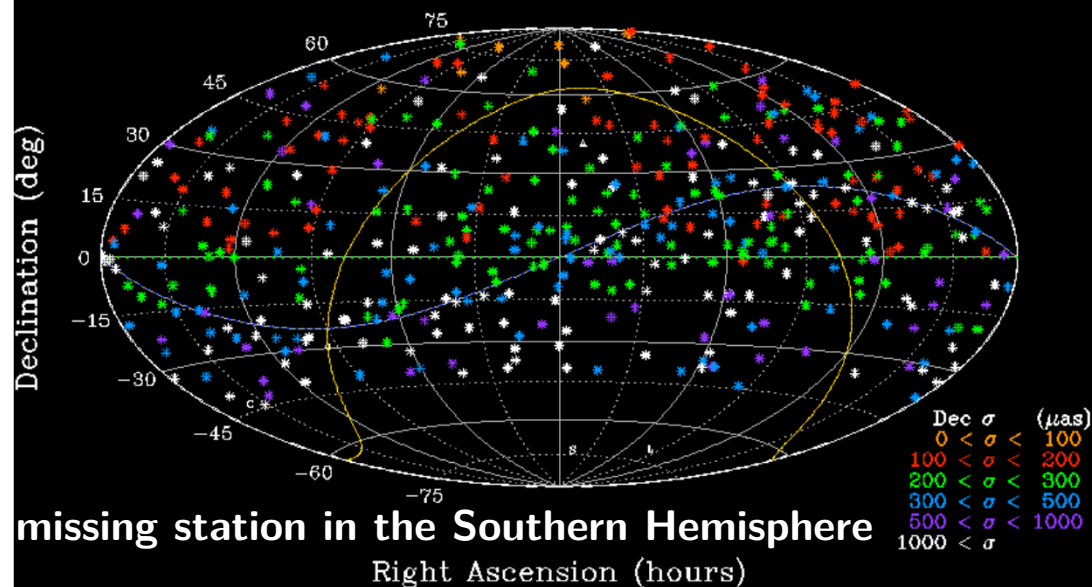
## ICRF2 S/X-Band (8.4 GHz)



## ICRF at X/Ka-Band (32 GHz)

469 Sources

Distribution of 436 Sources



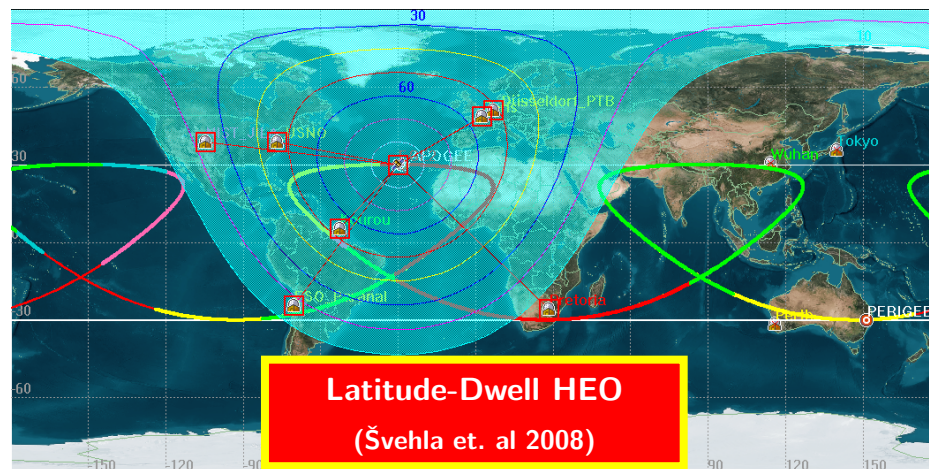
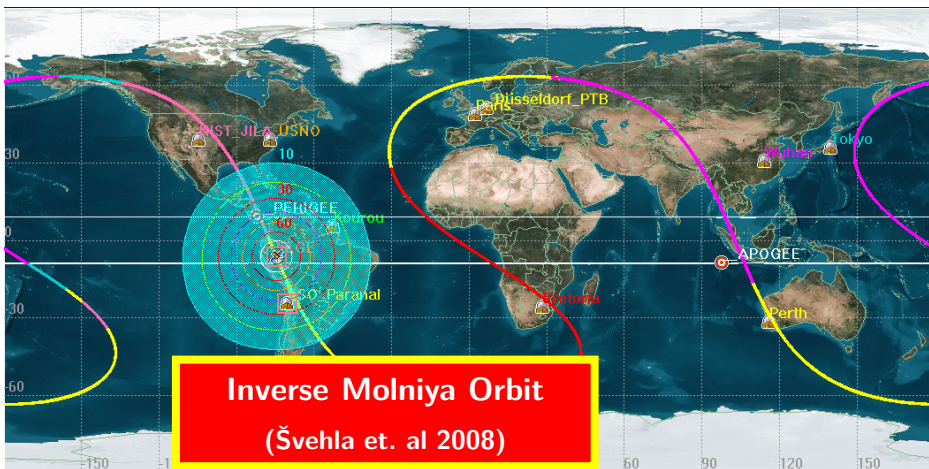
## ICRF2 S/X-Band

- # defining sources: 295
- # total sources: 3414 (3119 compact sources)
- Noise floor:  $\sim 40 \mu\text{as}$ , 5-6 $\times$  better than ICRF1
- Axis stability:  $\sim 10 \mu\text{as}$ , 2 $\times$  better than ICRF1 (IERS Conventions 2010)

(used sampling rate 112 Mbps at X/Ka-Band)

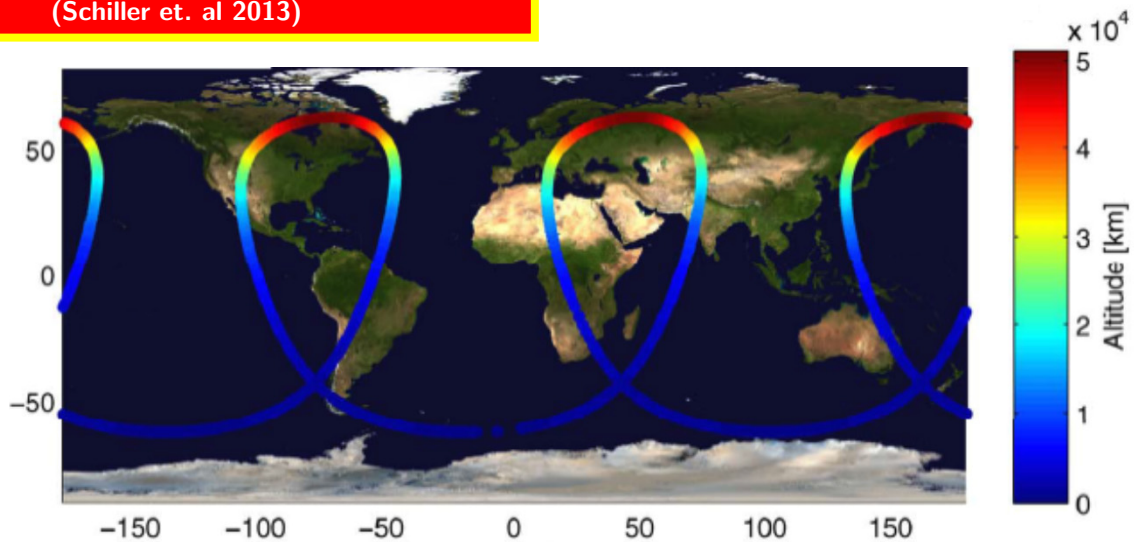
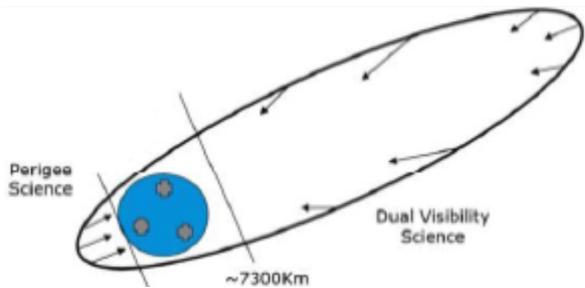
missing station in the Southern Hemisphere

# Orbit Design: Space-Time Explorer (STE-QUEST Misija)



**Finalbi Orbit Design (Latitude-Dwell HEO)**  
(Schiller et. al 2013)

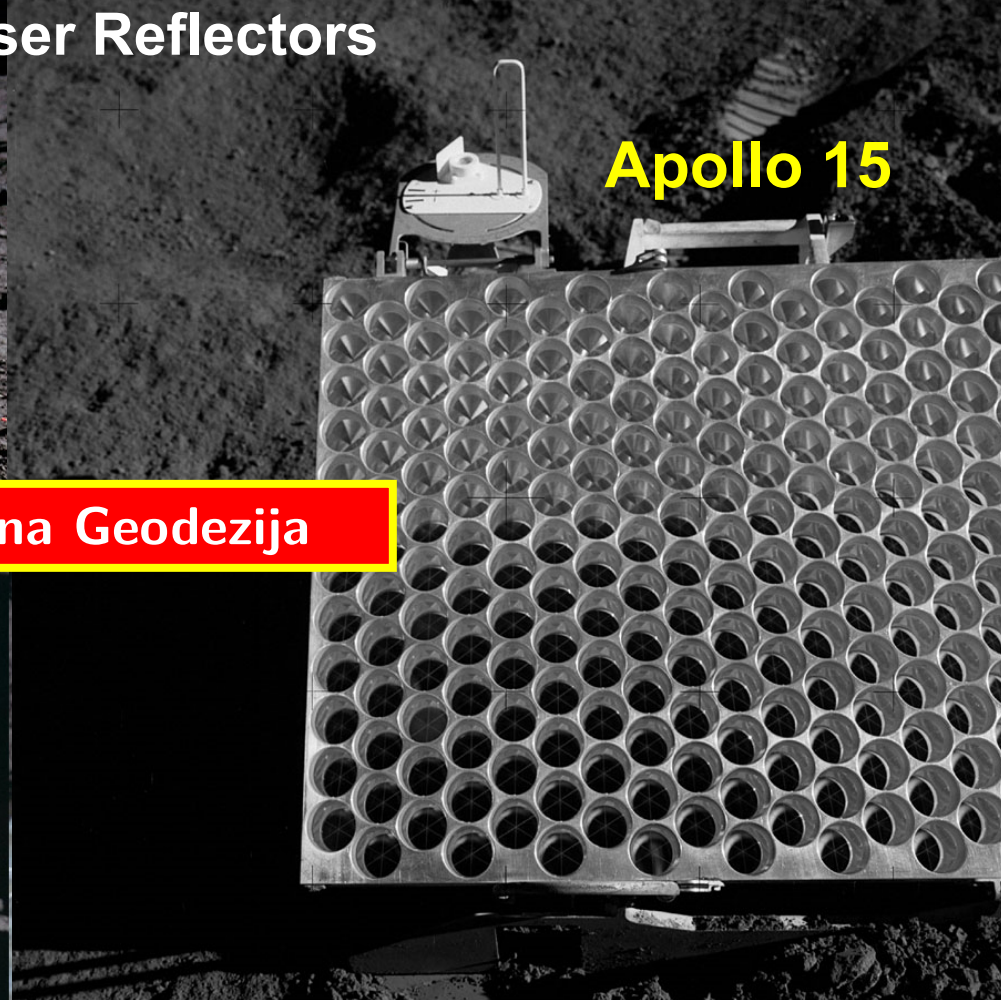
Apogee: 30-50 000 km  
Perigee: 1500-2200 km



# Lunar Laser Reflectors

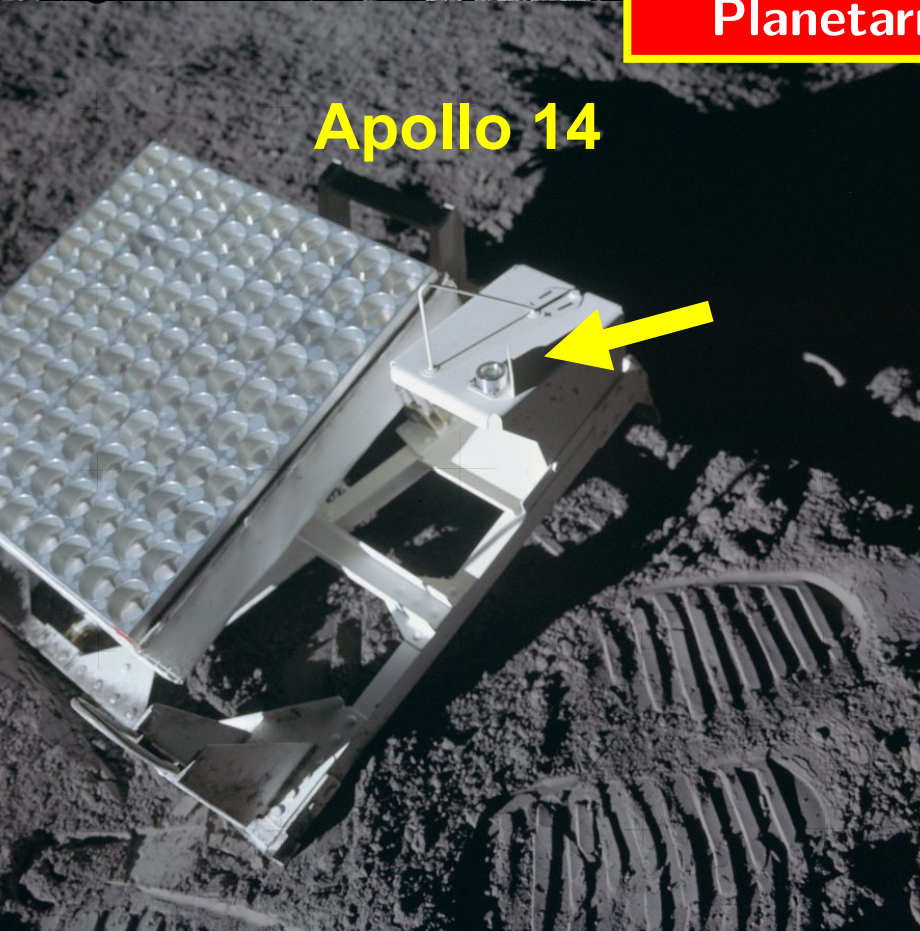


**Apollo 11**



**Apollo 15**

**Planetarna Geodezija**



**Apollo 14**

**Open Reflector**



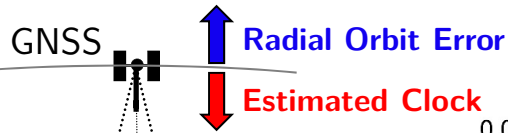


# Galileo Rezidualni Parametri Satova Satelita Nakon $J_2$ -Relativističke Korekcije

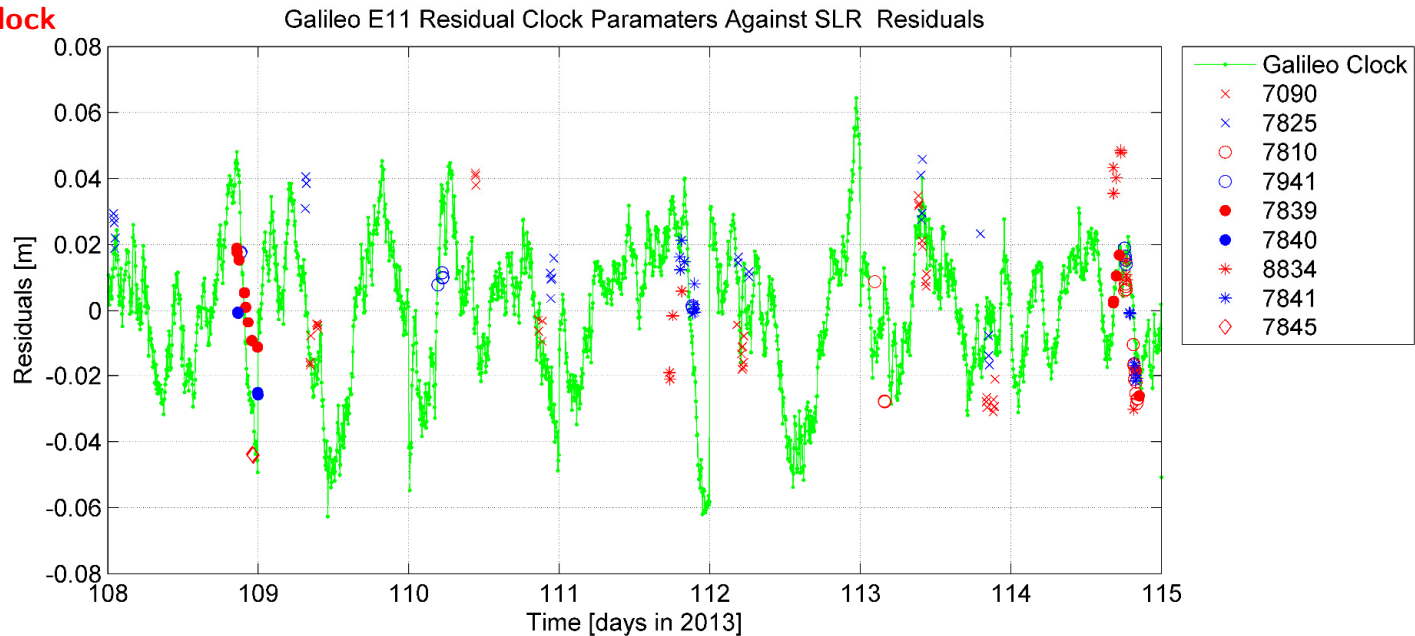
Ekvivalencija između  
Orbite i Sata:

$$\text{Sat} = - \text{Radijalna Pogreška}$$

Galileo H-maser precizniji nego SLR!

GNSS  
  
 Radial Orbit Error  
 Estimated Clock

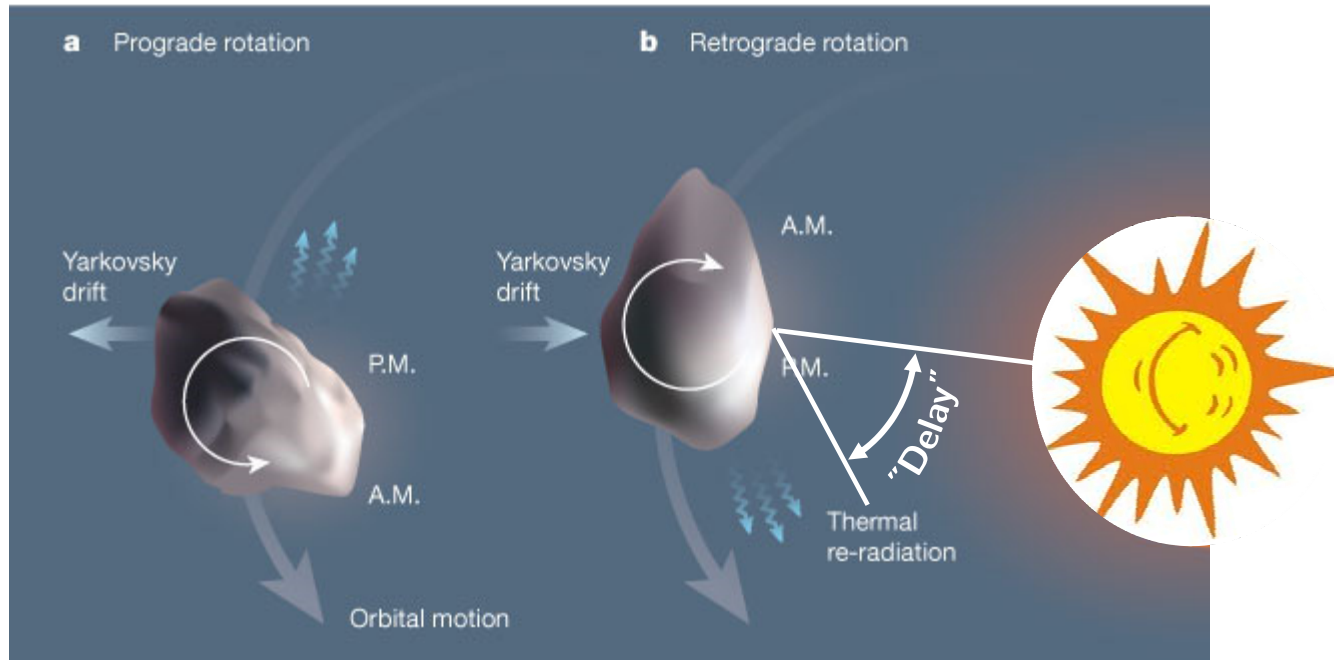
max.  $\approx 12^\circ$   
Galileo



Simulated Galileo H-Maser (mm)

N	0.2 h	0.25 h	0.5 h	1.0 h	1.5 h	6 h	12 h	14 h	24 h
1	1.2	1.4	2.0	2.7	3.4	6.8	9.3	11.2	<u>15.5</u>
2	1.0	1.1	1.5	2.2	2.7	5.7	7.7	8.8	10.3
3	0.8	0.9	1.3	1.9	2.3	4.7	6.5	7.8	9.8
4	0.7	0.8	1.2	1.7	2.1	4.3	5.8	6.6	8.7
5	0.8	0.9	1.1	1.5	1.9	3.8	5.2	5.6	7.8

# Termalna Inercija Illuminated Galileo Satelita - Yarkovsky Effect

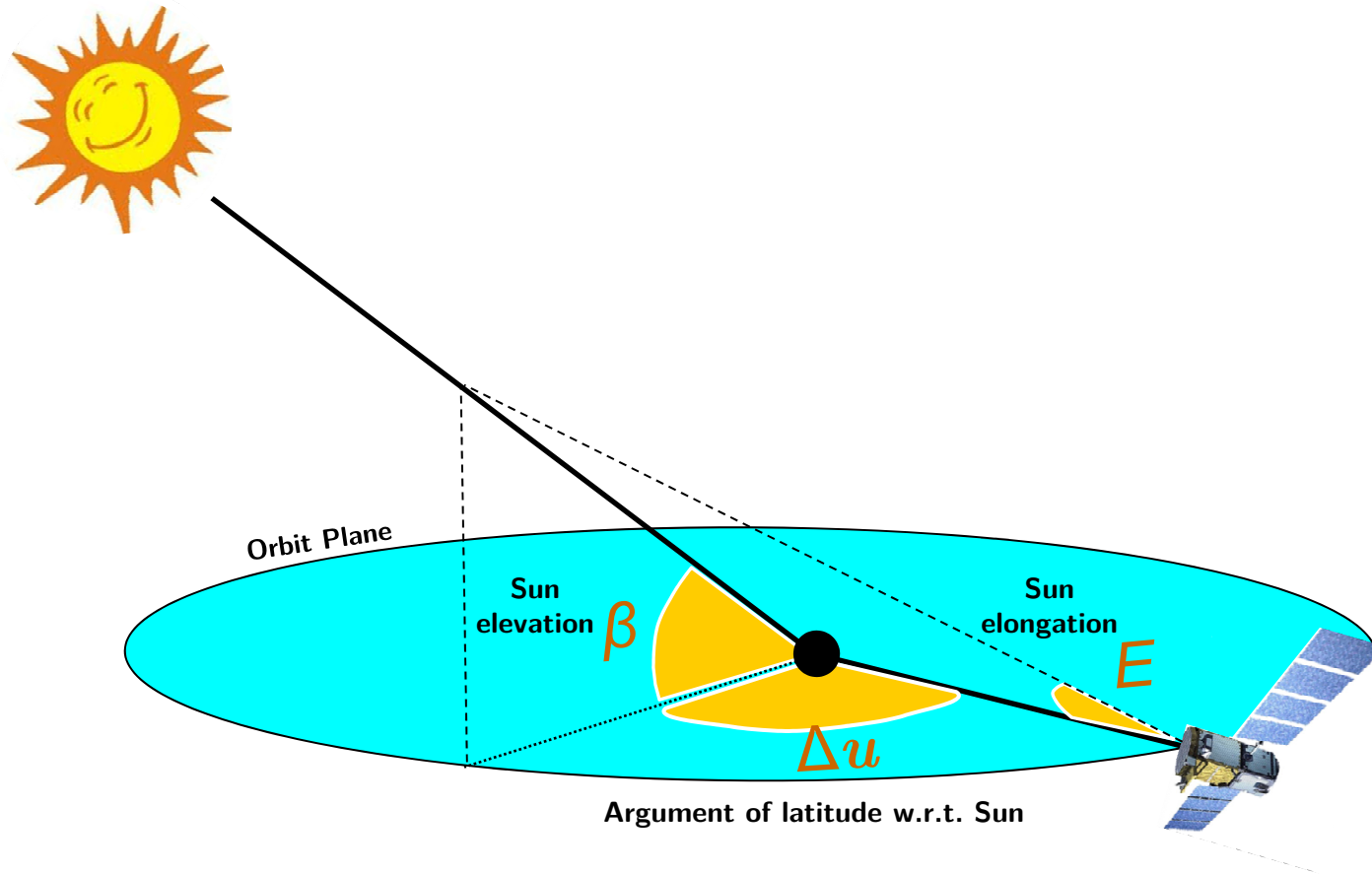


Richard P. Binzel, *Nature* **425**, 131-132 (2003)  
doi:10.1038/425131a

“An asteroid is warmed by sunlight and its “afternoon” side becoming hottest. As a result, that face of the asteroid re-radiates most thermal radiation, creating a recoil force on the asteroid and causing it to drift a little.

**The direction of the radiation depends on whether the asteroid is rotating in a prograde (anticlockwise) manner (a) or in a retrograde (clockwise) manner (b).”**

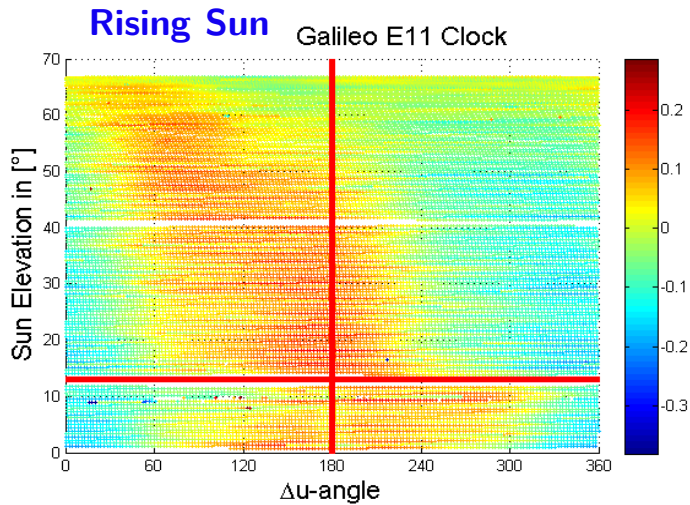
# Sun-Fixed Referentni Koordinatni Sustav



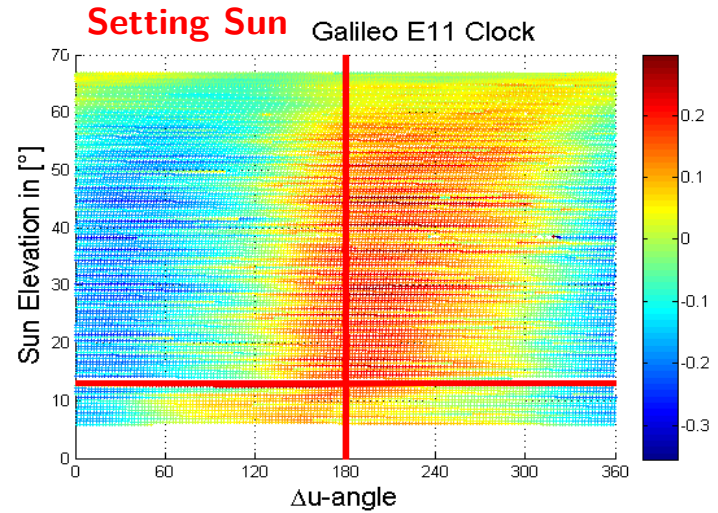
$$\cos E = -\cos \beta \cos \Delta u$$

Sun Elongation Angle

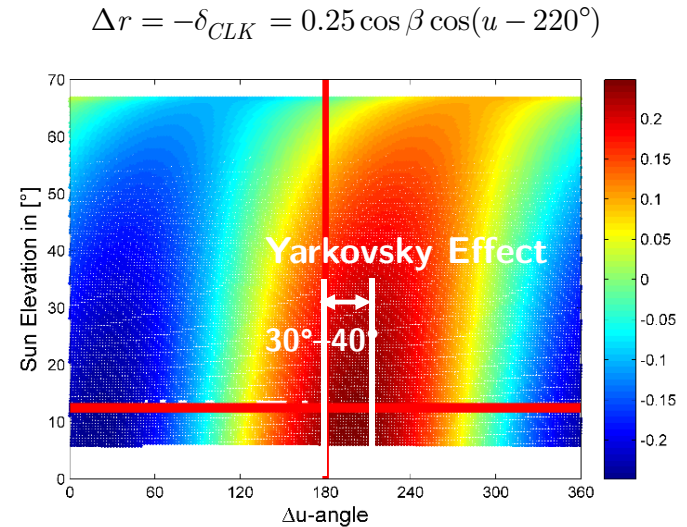
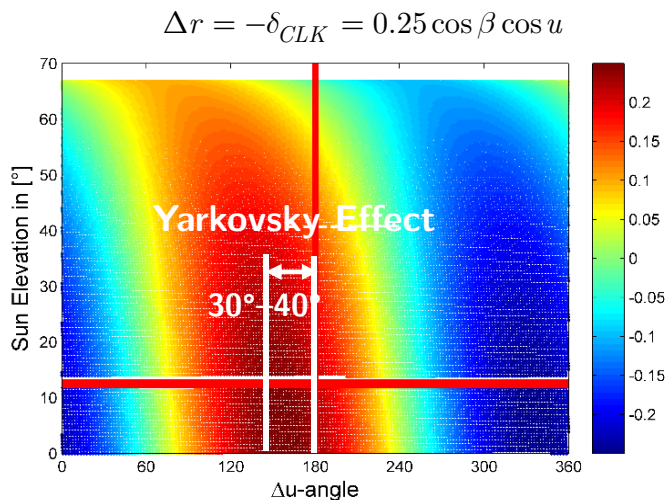
# Galileo Satovi Satelita i Yarkovsky Efekt



**Rising Sun**

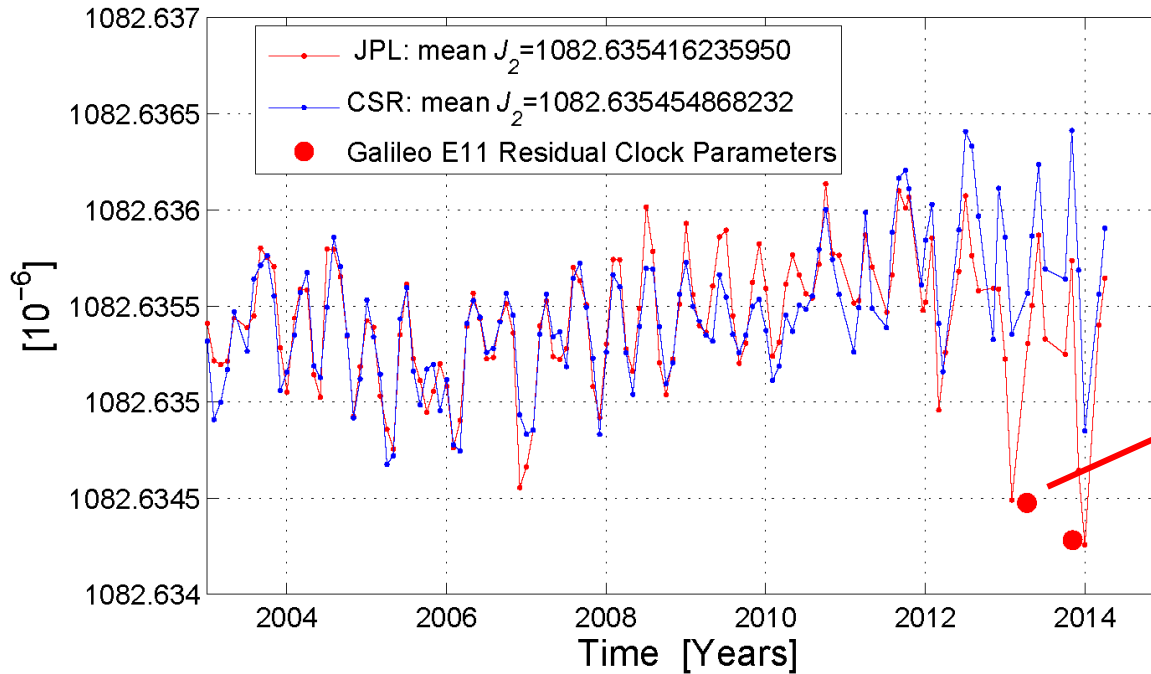


**Setting Sun**



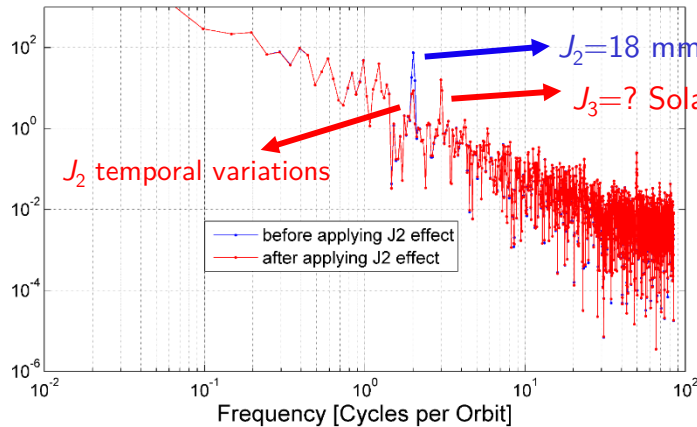
# Galileo H-maser: Određivanje $J_2$ Koeficijenta Gravitacijskog Polja Zemlje

Temporal Variations:  $J_2$  from GRACE RL05



$J_2$  prvi geometrijski rezultat baziran na circularnom modelu orbite

PSD of residual clock parameters 300-311/2013



$$\Delta t(J_2) = -\frac{3}{2} \frac{a_E^2}{a^2 c^2} J_2 \sqrt{GMa} \cdot \sin^2 i \sin 2u$$

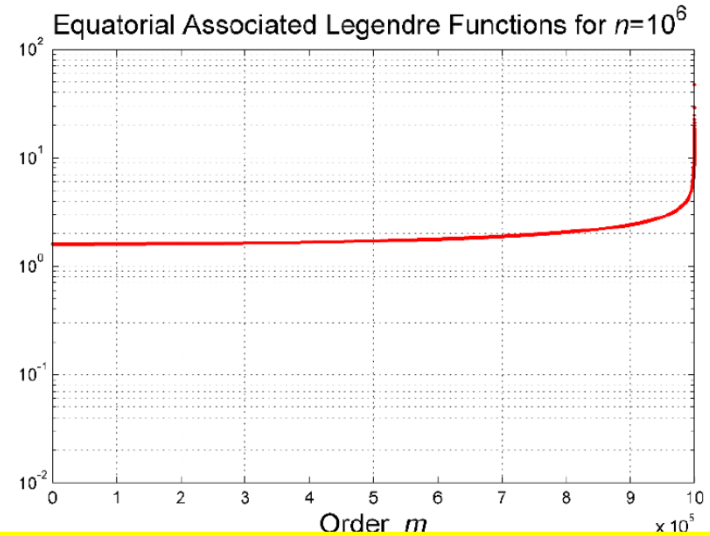
# Geometrijska Metoda Računanja Sfernih Harmonika, Vremenskih Promjena Sile Teže i Orbita Satelita

## Rotacija Sfernih Harmonika "Ortodroma u hyper-prostoru"

$$\begin{pmatrix} S_{n_{\max}m}(\alpha) \\ \dots \\ S_{mm}(\alpha) \\ C_{mm}(\alpha) \\ \dots \\ C_{n_{\max}m}(\alpha) \end{pmatrix} := \begin{pmatrix} S_{n_{\max}m} \\ \dots \\ S_{mm} \\ C_{mm} \\ \dots \\ C_{n_{\max}m} \end{pmatrix} \cos m\alpha + \begin{pmatrix} C_{n_{\max}m} \\ \dots \\ C_{mm} \\ -S_{mm} \\ \dots \\ -S_{n_{\max}m} \end{pmatrix} \sin m\alpha$$

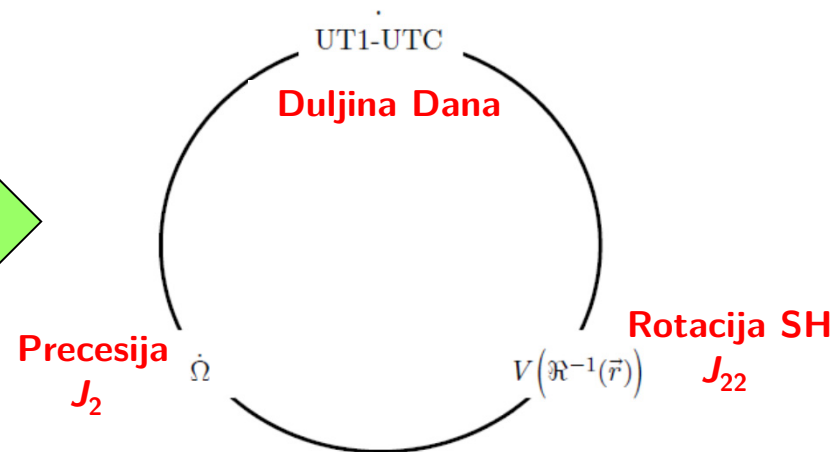
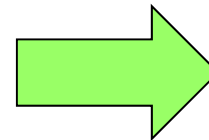
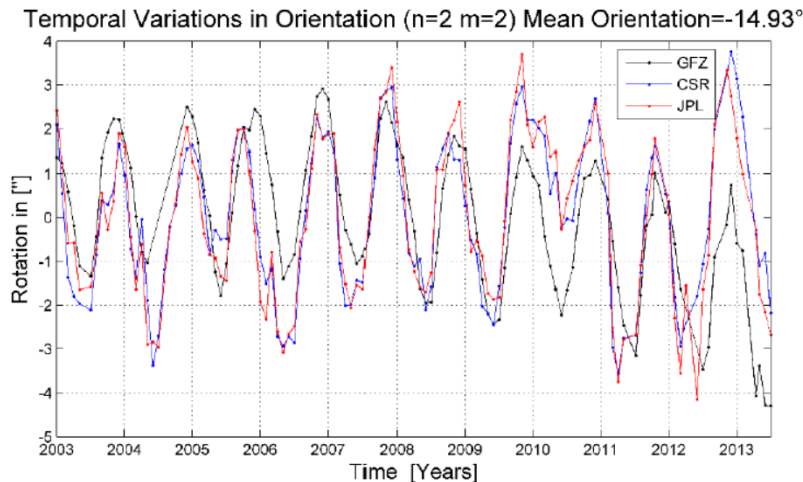
$$\hat{\Lambda}_m(\alpha) := \Lambda_m \cos m\alpha + \Lambda_m^* \sin m\alpha$$

## Računanje Sfernih Harmonika u Razvoju do Reda $10^6$



## Rotacija Tri-Aksijalnog Zemljinog Elipsoida

## Ekvivalentnost između Precesije, Duljine Dana i Rotacije Sfernih Harmonika



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**PUNO HVALA Prof. Nikoli Solarić i Prof. Miljenku Solarić  
za sve svoje znanje koje su prenijeli generacijama nas studenata!**

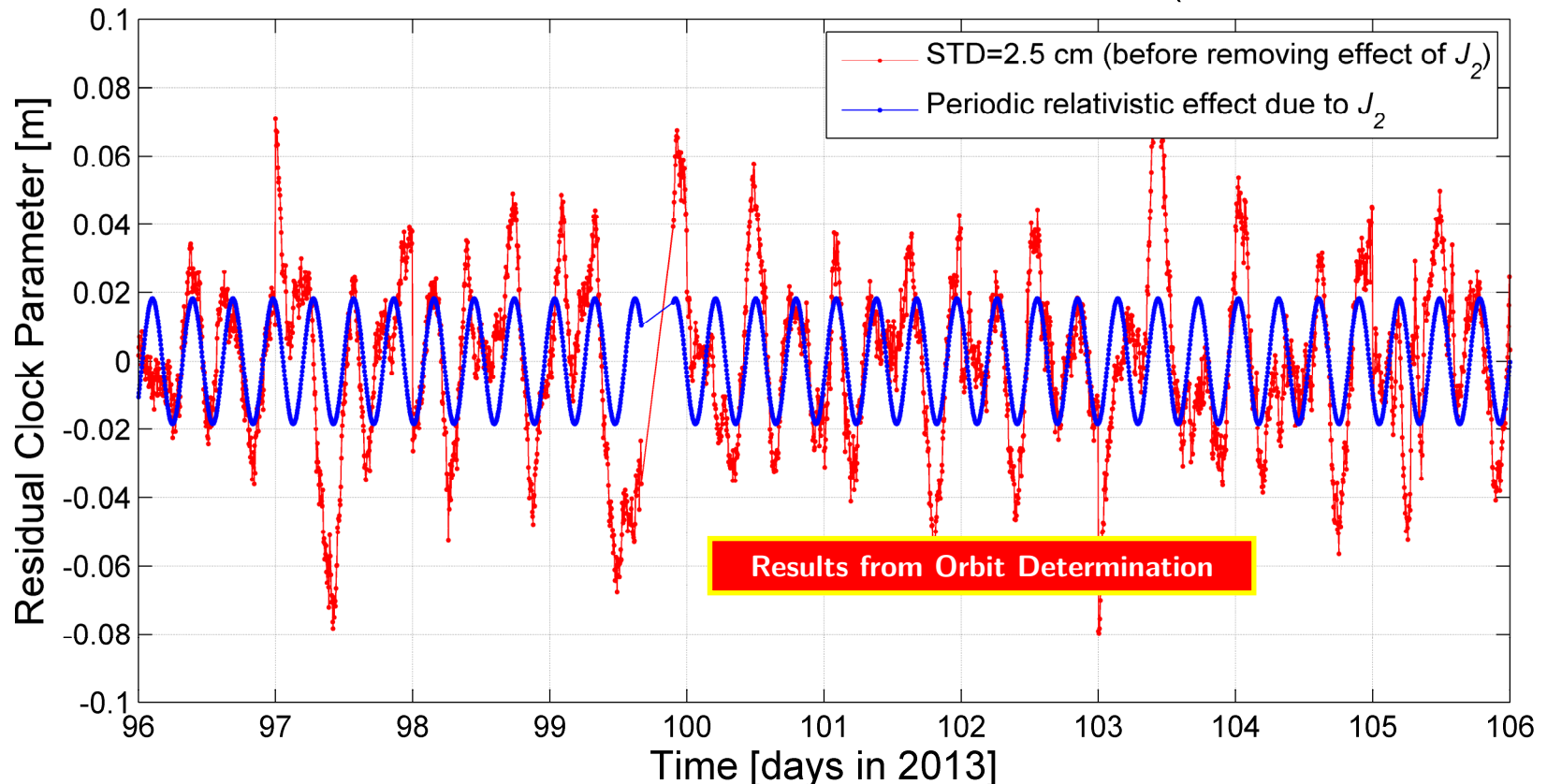
# $J_2$ Periodic Relativistic Effect in the Estimated Galileo Clock Parameters (H-Maser)

$$\Delta t(J_2) = -\frac{3}{2} \frac{a_E^2}{a^2 c^2} J_2 \sqrt{GMa} \cdot \sin^2 i \sin 2u$$

Galileo residual clock parameters (Clock Phase)  
estimated every 300 s over 10 days

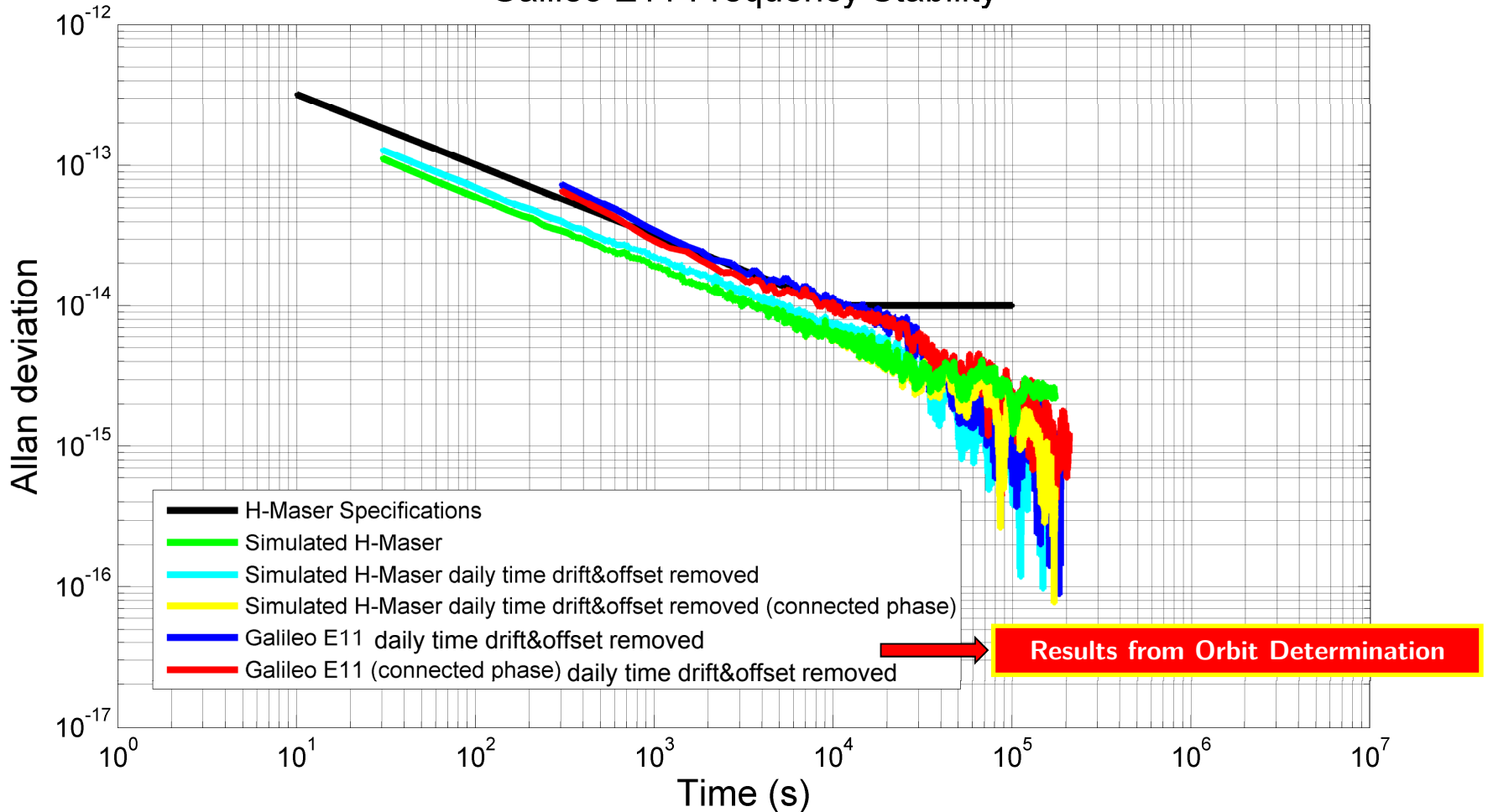
daily time bias and time drift removed

Galileo E11 Residuals Clock Parameters Without  $J_2$  Correction (Sun Elevations  $60^\circ$ - $65^\circ$ )





# Galileo E11 Frequency Stability



		Simulated Galileo H-Maser (mm)								
N		0.2 h	0.25 h	0.5 h	1.0 h	1.5 h	6 h	12 h	14 h	24 h
1		1.2	1.4	2.0	2.7	3.4	6.8	9.3	11.2	<u>15.5</u>
2		1.0	1.1	1.5	2.2	2.7	5.7	7.7	8.8	10.3
3		0.8	0.9	1.3	1.9	2.3	4.7	6.5	7.8	9.8
4		0.7	0.8	1.2	1.7	2.1	4.3	5.8	6.6	8.7
5		0.8	0.9	1.1	1.5	1.9	3.8	5.2	5.6	7.8

N=1: daily time bias and drift removed

# Galileo $J_2$ Radial Orbit Amplitude due to Temporal Gravity Variations

